New College Bradford
A-Level Further Mathematics
Y12-13
Summer Independent Learning

	Content	Answers
Compulsory		
Consolidation	Vectors Matrices Collisions	Vectors Matrices Collisions
Preview	Year 13 Content	
Optional		
Extension	Optional Extension	

Please bring all your SIL with you to your first lesson of Y13 Further Maths ☺

Vectors

Question 1

(a) Two planes with equations $\mathbf{r}.\begin{bmatrix}3\\1\\k\end{bmatrix}=5-3k$ and $\mathbf{r}.\begin{bmatrix}2\\5\\-k\end{bmatrix}=12+3k$ intersect in the line L.

Find a direction vector for the line L.

[2 marks]

(b) Show that the point (1, 2, −3) lies in both planes, and write down a vector equation for the line L.

[3 marks]

Question 2

Find the Cartesian equation of the plane which passes through the point (3, −4, 1) and which is parallel to the plane containing the line $x=y=z$ and the point (1, 2, −1).

[5 marks]

Question 3

The point $A(5, 2, 3)$ lies in the plane Π_1 with equation $\mathbf{r}.\begin{bmatrix}1\\3\\-2\end{bmatrix}=5$ **r**

Plane Π_2 has equation $32x-9y-22z=-71$.

The normal to Π_1 at A intersects Π_2 at B.

Find the perpendicular distance from B to Π_1.

[6 marks]

Matrices

Question 1

(a) The matrix **A** is defined by $\mathbf{A} = \begin{bmatrix} x & x^2 & 1 \\ x^2 & x & 1 \\ x^3 & x^3 & 1 \end{bmatrix}$.

Calculate the determinant of **A**, giving your answer in a fully factorised and simplified form.

[4 marks]

(b) Given that det(**B**) = $2(x - 1)$, find a fully factorised expression for det(**AB**).

[1 mark]

Question 2

Find the possible values of x, given that the following matrix is singular:

$$\begin{bmatrix} 1-x & 1 & -2 \\ -1 & 2-x & 1 \\ 0 & 1 & -1-x \end{bmatrix}$$

[4 marks]

Question 3

$\mathbf{P} = \begin{bmatrix} -2 & x & -3 \\ 3 & -x & 5 \\ 1 & 2 & 2 \end{bmatrix}$ and $\mathbf{R} = \begin{bmatrix} 2 & -1 & -2 \\ -1 & 2 & 2 \\ 2 & 3 & 3 \end{bmatrix}$.

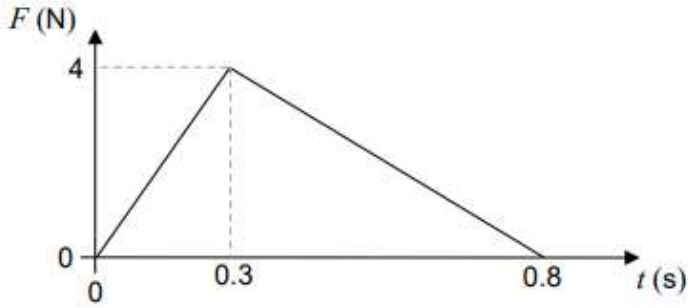
Given that **PQ** = **R**, calculate **Q**.

[7 marks]

Collisions

Question 1

The graph shows how a force, F, varies with time during a period of 0.8 seconds.



Find the magnitude of the impulse of F during the 0.8 seconds.

Circle your answer.

[1 mark]

1.0 Ns 1.6 Ns 2.2 Ns 3.2 Ns

Question 2

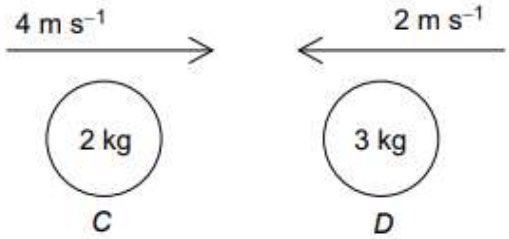
Two small smooth discs, C and D, have equal radii and masses of 2 kg and 3 kg respectively.

The discs are sliding on a smooth horizontal surface towards each other and collide directly.

Disc C has speed 4 m s^{-1} and disc D has speed 2 m s^{-1} as they collide.

The coefficient of restitution between C and D is 0.6

The diagram shows the discs, viewed from above, before the collision.



(a) Show that the speed of D immediately after the collision is 1.8 m s^{-1}, correct to 2 significant figures.

[4 marks]

(b) Find the speed of C immediately after the collision.

[2 marks]

(c) In fact the horizontal surface on which the discs are sliding is not smooth.

Explain how the introduction of friction will affect your answer to part **(b)**.

[2 marks]

Question 3

A disc, of mass 0.15 kg, slides across a smooth horizontal table and collides with a vertical wall which is perpendicular to the path of the disc.

The disc is in contact with the wall for 0.02 seconds and then rebounds.

A possible model for the force, F newtons, exerted on the disc by the wall, whilst in contact, is given by

$$F = kt^2(t-b)^2 \quad \text{for} \quad 0 \le t \le 0.020$$

where k and b are constants.

The force is initially zero and becomes zero again as the disc loses contact with the wall.

(a) State the value of b.

[1 mark]

(b) Find the magnitude of the impulse on the disc, giving your answer in terms of k.

[3 marks]

(c) The disc is travelling at 4 m s^{-1} when it hits the wall.

The disc rebounds with a speed of 2 m s^{-1}

Find k.

[3 marks]

Question 4

A lump of clay has mass 4 kg.

It is thrown so that it hits a vertical wall with a speed of 3 m s^{-1} and comes to rest, stuck to the wall.

It is assumed that the lump of clay takes 0.5 seconds to come to rest.

A possible model for the magnitude of the force, F newtons, exerted by the wall on the lump of clay is $F = at(2t-1)^2$ for $0 \le t \le 0.5$

(a) Find the value of a.

[5 marks]

(b) Find the kinetic energy lost as the lump of clay is brought to rest.

[2 marks]

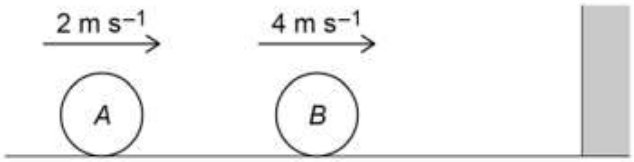
Question 5

Two spheres, *A* and *B*, of equal radii, move along a straight line on a smooth horizontal surface, in the same direction.

A vertical wall is perpendicular to the path of the spheres.

A moves at 2 m s^{-1} and *B* moves at 4 m s^{-1}.

The mass of *A* is 3 kg and the mass of *B* is 1 kg.



(a) The coefficient of restitution between sphere *B* and the wall is 0.75.

Find the speed of *B* after it collides with the wall.

[1 mark]

(b) The two spheres then collide.

The coefficient of restitution between the spheres is e.

(b) (i) Show that the velocity of A after the collision is $\frac{1}{4}(3-5e)$.

[4 marks]

(b) (ii) Find, in terms of e, the velocity of *B* after the collision.

[2 marks]

(b) (iii) If the direction of *A* is changed during the collision, what can be deduced about the value of e?

[2 marks]

Question 6

A ball of mass 0.3 kg is moving in a straight line with speed 4 m s^{-1}.

The ball collides with a wall perpendicular to its direction of motion and rebounds with speed 1 m s^{-1}.

Find the magnitude of the impulse, in Ns, exerted by the wall on the ball.

Circle your answer

[1 mark]

0.9 1.5 2.25 2.55

Question 7

Two smooth discs *A* and *B* have equal radii and are free to move on a smooth horizontal surface.

A and *B* have masses 4 kg and 2 kg respectively.

A moves with speed 3 m s^{-1} in a straight line and *B* moves in the same direction with speed 2 ms^{-1}.

A and *B* collide; the coefficient of restitution between *A* and *B* is e

(a) (i) Show that the speed of *A* immediately after the collision is

$$\frac{8-e}{3} \text{ m s}^{-1}$$

[4 marks]

(a) (ii) Find the speed of *B* immediately after the collision, in terms of e

[2 marks]

(b) The speed of *B* after the collision is k times the speed of *A*.

Find the maximum value of k

Fully justify your answer.

[3 marks]

Preview

Watch the vidoes by clicking on the link below. Make flashcards to help with the new knowledge. This will help you get a better headstart for next year.

De Moivre's Theorem
Euler's Form for Complex Numbers
Complex Roots of Unity

Use the AQA Spec document on OneNote to do some reading ahead about which topics are coming up next year.

John will love it if you have a list of questions to ask him when you return in September ☺

Optional Extension

Have a look at the “Further Risps” using:

http://www.s253053503.websitehome.co.uk/further-risps/further-risps-20-4-20.pdf

Try to expand your mathematical knowledge by trying at least **5** of them.

Here are some recommended ones that you might find interesting:

Further Risp 1: The Twizzle

Further Risp 2: The Spiral-Line Area

Further Risp 4: Series Arithmagon

Further Risp 5: Inverse Trig Triangles

Further Risp 8: Induction Number Theory

Further Risp 9: The Unexpected Group

Further Risp 12: Sketching Rational Functions

Further Risp 14: Three Simultaneous Equations

Further Risp 20: Matrix Arithmagon

Further Risp 22: The Vector Product

Further Risp 25: Trace Arithmetic

Further Risp 34: The Series Result

Solutions

Vectors

Question 1

6 (a)	Direction of L is perpendicular to normal $\begin{bmatrix}3\\1\\k\end{bmatrix}\times\begin{bmatrix}2\\5\\-k\end{bmatrix}=\begin{bmatrix}-6k\\5k\\13\end{bmatrix}$	1 1
	Total	**2**
6 (b)	Substitute (1, 2, −3) into each equation. Correct RHS, therefore lies on both planes $L:\ \mathbf{r}=\begin{bmatrix}1\\2\\-3\end{bmatrix}+\lambda\begin{bmatrix}-6k\\5k\\13\end{bmatrix}$	1 1 1
	Total	**3**

Question 2

7	Obtain points in parallel plane, eg (0, 0, 0) and (1, 1, 1)	1
	Obtain 2 vectors in parallel plane, eg $\begin{bmatrix}1\\2\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\1\\1\end{bmatrix}$	1 1
	Vector product for normal vector $\begin{bmatrix}3\\-2\\-1\end{bmatrix}$	1 1
	Use with point in plane equation to obtain d $3x-2y-z=16$	
	Total	**5**

Question 3

8	Normal to Π_1 at A: $\mathbf{r}=\begin{bmatrix}5\\2\\3\end{bmatrix}+\lambda\begin{bmatrix}1\\3\\-2\end{bmatrix}$	1 1
	Substitute into Π_2: $\begin{bmatrix}5+\lambda\\2+3\lambda\\3-2\lambda\end{bmatrix}.\begin{bmatrix}32\\-9\\-22\end{bmatrix}=-71$	1 1
	$160+32\lambda-18-27\lambda-66+44\lambda=-71$ so $\lambda=-3$	1
	$\overline{OB}=\begin{bmatrix}2\\-7\\9\end{bmatrix}$	1
	$AB=\lvert\overline{AB}\rvert$ $=\left\lvert\begin{bmatrix}-3\\-9\\6\end{bmatrix}\right\rvert=3\sqrt{14}$	
	Total	**6**

Matrices

Question 1

5 (a)	Clear evidence of row/column operation (allow 1 error) Clear evidence of 2nd row/column operation (allow 1 error) Extraction of one factor det = $x^2(x-1)^2(2x+1)$	1 1 1 1
	Total	**4**
5 (b)	Use of det(**AB**) = det(**A**) det(**B**) det(**AB**) = $2x^2(x-1)^3(2x+1)$	1ft
	Total	**1**

Question 2

6	singular implies determinant = 0 Attempts to find determinant, either row/col operations or expansion det = $(x-1)(x+1)(2-x)$ x = 1, 2, –1	1 1 1 1ft
	Total	**4**

Question 3

7	**PQ**=**R** implies **Q**=**P**$^{-1}$**R** det**P** = 2 Method to find **P**$^{-1}$ (minors/cofactors – 3 or more correct) $\mathbf{P}^{-1}=\frac{1}{2}\begin{bmatrix}-2x-10 & -2x-6 & 2x\\ -1 & -1 & 1\\ x+6 & x+4 & -x\end{bmatrix}$ **Q**= **P**$^{-1}$**R** evidence of multiplication (ft incorrect order) $\mathbf{Q}=\frac{1}{2}\begin{bmatrix}2x-14 & 4x-2 & 6x+8\\ 1 & 2 & 3\\ 8-x & -2x+2 & -3x-4\end{bmatrix}$	1 1 1 1,1 1 1
	Total	**7**

Collisions

Question 1

2	Circles correct answer.	AO1.1b	B1	$I = \frac{1}{2} \times 0.8 \times 4 = 1.6\,\text{Ns}$

Question 2

5(a)	Forms an equation using conservation of momentum.	AO1.1a	M1	CoM $2 \times 4 - 2 \times 3 = 2v_C + 3v_D$
	Forms an equation using coefficient of restitution.	AO1.1a	M1	$2v_C + 3v_D = 2$ Newton's law of restitution
	Obtains two correct equations.	AO1.1b	A1	$v_C - v_D = -0.6(-2-4)$
	Completes a rigorous argument using both conservation of energy and the coefficient of restitution to find speed of D to the specified accuracy. Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	$v_C - v_D = -3.6$ $5v_D = 9.2$ $v_D = 1.84$ $= 1.8\ \text{m s}^{-1}$ to 2 sf
5(b)	Forms equation to find velocity of C	AO1.1a	M1	$1.84 - v_C = 3.6$ $v_C = -1.76$ Speed of $C = 1.8\ \text{m s}^{-1}$ to 2 sf
	Obtains correct speed for C.	AO1.1b	A1	
5(c)	Gives a valid explanation (eg collision is instantaneous, no distance travelled, no work done, no energy lost to friction during collision, etc)	AO2.4	E1	The introduction of friction will not affect my answer to **(b)** because the collision is instantaneous.
	Therefore answer to part **(b)** is not affected by the introduction of friction. (depends on E1 above)	AO2.2a	R1	
	Total		**8**	

Question 3

7(a)	Deduces correct value for b.	AO2.2a	B1	$b = 0.02$
7(b)	Forms an integral to find the impulse.	AO3.4	M1	$I = \int_0^{0.02} kt^2(t-0.02)^2\,\mathrm{d}t$
	Integrates terms and uses limits or uses a calculator for definite integral (PI)	AO1.1a	M1	$= k\int_0^{0.02} (t^4 - 0.04t^3 + 0.0004t^2)\,\mathrm{d}t$
	Obtains correct value for impulse. (AWRT 1.1×10^{-10})	AO1.1b	A1	$= k\left[\frac{t^5}{5} - \frac{t^4}{100} + \frac{t^3}{7500}\right]_0^{0.02}$ $= k \times 1.07 \times 10^{-10}$ Ns
7(c)	Uses 'impulse equals change in momentum' to form an equation, with 'their' impulse from **(a)**.	AO3.4	M1	$k \times 1.07 \times 10^{-10} = \lvert 0.15 \times 4 - 0.15 \times (-2) \rvert$
	Obtains a correct equation for 'their' impulse.	AO1.1b	A1F	$k = \frac{0.9}{1.07 \times 10^{-10}}$
	Obtains the correct value for k. CAO	AO1.1b	A1	$= 8.4 \times 10^9$
	Total		**7**	

Question 4

6 (a)	Forms an integral to find the impulse	AO3.4	M1	$\int_0^{0.5} at(2t-1)^2 \mathrm{d}t$ $= a\int_0^{0.5} \left(4t^3 - 4t^2 + t\right) \mathrm{d}t$ $= \frac{a}{48}$ $4 \times 3 = \frac{a}{48}$ $a = 576$
	Correctly evaluates 'their' integral	AO1.1b	A1	
	Uses $I = mv - mu$	AO1.1a	M1	
	Uses impulse to form an equation to find a	AO3.4	M1	
	Obtains correct value for a	AO1.1b	A1	
	Total		**5**	
6 (b)	Uses KE formula	AO1.1a	M1	KE lost $= \frac{1}{2} \times 4 \times 3^2$ $= 18$ J
	Obtains correct energy	AO1.1b	A1	
	Total		**2**	

Question 5

Q	Marking instructions	AO	Marks	Typical solution
8 (a)	Uses law of restitution	AO1.1b	B1	$v = 0.75 \times 4 = 3$
	Total		**1**	
8 (b)(i)	Applies principle of conservation of momentum	AO1.1a	M1	$3 \times 2 + 1 \times (-3) = 3v_A + v_B$ $3 = 3v_A + v_B$ $v_A - v_B = -e(2-(-3))$ $v_A - v_B = -5e$ $v_B = v_A + 5e$ $3 = 3v_A + v_A + 5e$ $v_A = \frac{1}{4}(3-5e)$
	Applies Newton's law of restitution	AO1.1a	M1	
	Solves 'their' equations to find the velocity for *A*	AO1.1a	M1	
	Obtains correct velocity for *B*	AO2.1	A1	
	Total		**4**	
8 (b)(ii)	Substitutes velocity of *A* to find velocity *B*	AO1.1a	M1	$v_B = \frac{1}{4}(3-5e)+5e$ $v_B = \frac{3}{4}(1+5e)$
	Obtains correct velocity for *A*	AO1.1b	A1	
	Total		**2**	
8 (b)(iii)	Forms inequality based on velocity of *A*	AO2.2a	M1	$3-5e<0$ $e > \frac{3}{5}$
	Obtains correct inequality for e	AO1.1b	A1	
	Total		**2**	

Question 6

1	Circles correct answer.	AO1.1b	B1	1.5

Question 7

8(a)(i)	Forms a conservation of momentum equation.	AO1.1a	M1	$16 = 4v_A + 2v_B$
	Obtains a correct equation.	AO1.1b	A1	$v_B - v_A = e$
	Uses coefficient of restitution to create a second correct equation.	AO1.1b	B1	$16 = 4v_A + 2(v_A + e)$
	Completes a rigorous argument to obtain the correct expression for the speed of A.	AO2.1	R1	$16 = 6v_A + 2e$ $v_A = \frac{8-e}{3}$
8(a)(ii)	Substitutes the speed of A back into either of their equations, or eliminates v_A from their original equations.	AO1.1a	M1	$16 = 4(v_B - e) + 2v_B$ $16 = 6v_B - 4e$ $v_B = \frac{8+2e}{3}$
	Obtains correct speed for B.	AO1.1b	A1	
8(b)	Explains that k is a maximum when e is a maximum.	AO2.4	E1	$k = \frac{8+2e}{8-e}$ k is maximum when $e = 1$
	Forms an expression for k	AO1.1a	M1	
	Calculates correct value of k	AO1.1b	A1	$k = \frac{10}{7}$
	Total		**9**	