Summer Independent Learning:

A level Mathematics

Welcome to A level maths!

These activities have been designed to help you revise key GCSE algebra content and show you how it will be applied to slightly more complicated examples at A level.

If you can print this booklet to work on, please do; otherwise, take your own notes from the screen. The exercises should be completed on separate lined paper, showing full detailed methods.

There are 11 sections – 7 are **compulsory** and 4 are **optional**. The **optional** sections will be set as homework during the first half-term, so you can use the summer to get ahead! Some sections will only take 20 minutes; others may take up to an hour.

For each section you complete:

1. Watch the video (for every section except the first; full playlist here)
    a. Watch and copy down the GCSE-level examples.
    b. Pause the video and try the "your turn" column.
    c. Check your answers by un-pausing the video. If you got any wrong, you may need to rewind and watch the examples again.
    d. Repeat a-c for the A-level examples.
2. Try the exercise.
3. Mark and correct your answers in a different colour (the answers are at the end of this document). For any you got wrong, write down why you got the wrong answer.
4. Make a note of any questions you need to ask in your first lesson.

On your first day with us in September:

- Bring an A4 ringbinder specifically for maths.
- Bring all the maths summer work you have completed.
- Be ready to ask your questions and help answer each other's questions.

Your initial maths assessment will be made up of questions exactly like those in the **compulsory** sections – so prepare well and you should be able to get a very good mark!

Watch out for this symbol! It highlights an area where misconceptions and mistakes are common.

Contents

Algebra Basics

Section 1: Common mistakes - **Compulsory**

For each shaded expression, write down (i) the correct equivalent form and (ii) an explanation that would help someone who gets (i) wrong. It may help to substitute some numbers in.

Example:

1) $a(x+y) \equiv ax+ay$ because $2 \times (3+4)$ is $2 \times 7 = 14$, which is not the same as $2 \times 3 + 4 = 6 + 4 = 10$.

The wrong answers to this exercise are some of the most common mistakes made by students every year!
Notice how many of them involve brackets!

$a(x+y)$	
$ax+y$	$ax+ay$
$a(x \times y)$	
$ax \times y$	$ax \times ay$
$(x+y)^2$	
x^2+y^2	x^2+y^2+2xy
$(3x)^2$	
$9x^2$	$3x^2$
$(x+y)^3$	
$x^3+y^3+3x^2y+3xy^2$	x^3+y^3
$\left(\sqrt{x+y}\right)^2$	
$x+y$	$x+y+2\sqrt{x}\sqrt{y}$
$x+y$	
$\sqrt{x^2+y^2}$	$\sqrt{x^2}+\sqrt{y^2}$
$\sqrt{4x}$	
$4\sqrt{x}$	$2\sqrt{x}$
$9(x+y)^2$	
$(9x+9y)^2$	$(3x+3y)^2$
$\sqrt{9x+9y}$	
$9\sqrt{x+y}$	$3\sqrt{x+y}$
$\left(\frac{2}{x}\right)^3$	
$\frac{2}{x^3}$	$\frac{8}{x^3}$
$\frac{\sqrt{20}}{2}$	
$\sqrt{5}$	$\sqrt{10}$
2^{x+y}	
2^x+2^y	$2^x \times 2^y$
6^x	
3×2^x	$3^x \times 2^x$
$\sin(x+y)$	
$\sin(x)+\sin(y)$	$\sin(x)\cos(y)+\sin(y)\cos(x)$
$\sin(2x)$	
$2\sin(x)$	$2\sin(x)\cos(x)$

Section 2: Collecting terms - **Compulsory**

Terms are separated by + or – signs.

Brackets (and other groupings) make multiple terms behave like a single term.

Video: https://youtu.be/d_shzeTUkGc

Examples	**Your turn**
At GCSE:	
Simplify by collecting terms  $4x^2 + 7x - 1 - (2x^2 - 9x + 3)$	Simplify by collecting terms $3x^2 - 4x + 10 - (3x - x^2 - 12)$
At A level:	
Simplify by collecting terms $4(x+3)^2 + 7\sin(x) - 2^x - (2(x+3)^2 - 9\sin(x) + 3 \times 2^x - 6^x)$	Simplify by collecting terms $3(x-2)^2 - 4x + 10\cos(x) - (3x - (x-2)^2 - 12\cos(x))$

Simplify by collecting terms

1. $4x^4 - 3x^3 + 7x^2 - 3 - (3x^4 + 2x^3 - 7x^2 - 9)$
2. $4(x+3)^2 - 2(x+1)^2 - 7(x+3) - 2(x+1)^2$
3. $7\sin(x) + 2\sin(2x) + 3\cos(2x) - 4\sin(x) - 8\sin(2x)$
4. $3^x + 3^x - 3 \times 3^x + 9^x + 3 \times 9^x$
5. $(x+1)^2 + (x+1) + x^2 - (x+1)^2 - x - 1$

Section 3: Expanding brackets - **Optional**

Video: https://youtu.be/dJ8BkO7VZVc

Examples	**Your turn**
At GCSE:	
Expand $(x+7)(2x-3)$	Expand $(x-3)(3x+1)$
At A level:	
Expand $(x+7)(2x-3)(4-x)$ $(x+7+3y)(2x-3-y)$	Expand $(x-3)(3x+1)(5-x)$ $(x^2-3x-1)(x^2+2x-3)$

Giving your answers in descending powers of x, expand and simplify

a $(x+1)(x^2+5x-6)$

b $(2x-5)(x^2-3x+7)$

c $(4-7x)(2+5x-x^2)$

d $(3x-2)^3$

e $(x^2+3)(2x^2-x+9)$

f $(4x-1)(x^4-3x^2+5x+2)$

g $(x^2+2x+5)(x^2+3x+1)$

h $(x^2+x-3)(2x^2-x+4)$

i $(3x^2-5x+2)(2x^2-4x-8)$

j $(x^2+2x-6)^2$

Section 4: Solving equations by factorising - **Optional**

Video: https://youtu.be/4zj2dbQxPYc

Examples	**Your turn**
At GCSE:	
Factorise $35a^6b^5 + 14a^5b^6$ Factorise, and hence solve $x^2 - 6x + 8 = 0$	Factorise $15a^3b^6 + 10a^4b^5$ Factorise, and hence solve $x^2 - 5x + 6 = 0$
At A level:	
Factorise $35(x+1)^6(x-2)^5 + 14(x+1)^5(x-2)^6$ Solve $35(x+1)^6(x-2)^5 + 14(x+1)^5(x-2)^6 = 0$ Solve $\frac{2^x(x+2)(x-3)^2}{(x+3)(x-1)} = 0$	Factorise $15(x-3)^3(x+4)^6 + 10(x-3)^4(x+4)^5$ Solve $15(x-3)^4(x+4)^6 + 10(x-3)^3(x+4)^5 = 0$ Solve $\frac{(x+3)(x-1)}{2^x(x+2)(x-3)^2} = 0$

Solve the following equations:

1. $(x+1)^2(x-3)+(x+1)(x-3)^2=0$
2. $(2x+1)^3-(x+4)(2x+1)^2=0$
3. $(3x-2)^4(2x+3)^3+(3x-2)^3(2x+3)^3=0$
4. $(1-x)^4(2-5x)^7-(1-x)^5(2-5x)^6=0$

a $\dfrac{(3-2x)(x+1)}{(x-3)(x+4)}=0$ **b** $\dfrac{(3-2x)(x+1)^2}{(x-3)(x+4)}=0$ **c** $\dfrac{x^2+x-6}{(x-3)(x+4)}=0$

d $\dfrac{(x^2-9)(x-1)}{(x-3)(x+4)}=0$ **e** $\dfrac{x^5(x-1)}{(x-3)(x+4)}=0$ **f** $\dfrac{\sin(x)(x-1)}{(x-3)(x+4)}=0$

Indices

Remember:

$$x^{-n} = \frac{1}{x^n}$$
$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

Section 1: Laws of indices - Compulsory

Video: https://youtu.be/TA1fWvZ3x08

Examples	Your turn
At GCSE:	
Express in the form ax^n:	Express in the form ax^n:
$6x^2 \times 3x^5$	$8x^3 \times 4x^7$
$\frac{6x^2}{3x^5}$	$\frac{8x^3}{4x^7}$
$(3x^5)^2$	$(4x^7)^3$
$\sqrt[3]{x^2}$	$\sqrt{x^3}$
At A level:	
Express in the form x^n:	Express in the form x^n:
$x^2 \times \sqrt{x}$ 	$x^3 \times \sqrt[3]{x}$
$\frac{\sqrt{x}}{\sqrt[3]{x}}$	$\frac{x^3}{\sqrt[3]{x}}$
$(x^2)^{-3}$ 	$(x^4)^{-4}$

Simplify

a $x^8 \times x^{-6}$ **b** $y^{-2} \times y^{-4}$ **c** $6p^3 \div 2p^7$ **d** $(2x^{-4})^3$

e $y^3 \times y^{-\frac{1}{2}}$ **f** $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$ **g** $x^{\frac{3}{5}} \div x^{\frac{1}{3}}$ **h** $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$ **j** $(3x^{\frac{2}{5}})^2$ **k** $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$ **l** $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

Express in the form x^k

a $\sqrt{x}$ **b** $\frac{1}{\sqrt[3]{x}}$ **c** $x^2 \times \sqrt{x}$ **d** $\frac{\sqrt[4]{x}}{x}$

e $\sqrt{x^3}$ **f** $\sqrt{x} \times \sqrt[3]{x}$ **g** $(\sqrt{x})^5$ **h** $\sqrt[3]{x^2} \times (\sqrt{x})^3$

Section 2: Single terms of the form ax^n - Compulsory

Video: https://youtu.be/h_AtTshkLuU

Write each term in the form ax^n

$\frac{1}{x^3}$	$\sqrt[3]{x}$	$\frac{3}{x^3}$	$\frac{1}{3x^3}$	$\frac{1}{3}x^3$

Express each of the following in the form ax^b, where a and b are rational constants.

a $\frac{4}{\sqrt{x}}$ **b** $\frac{1}{2x}$ **c** $\frac{3}{4x^3}$ **d** $\frac{1}{(3x)^2}$ **e** $\frac{2}{5\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt{9x^3}}$

Section 3: Multiple terms of the form ax^n - Compulsory

Video: https://youtu.be/_gZvq0lSjZQ

Examples	**Your turn**
At A level:	
Express as individual terms where possible: $\frac{x^2 + x\sqrt{x}}{\sqrt{x}}$ 	Express as individual terms where possible: $\frac{x}{x + x^2\sqrt{x}}$
$\frac{\sqrt{x}}{x^2 + x\sqrt{x}}$	$\frac{x + x^2\sqrt{x}}{x}$

Expand and simplify

a $x(x^2 - x^{-1})$ **b** $2x^3(x^{-1} + 3)$ **c** $x^{-1}(3x - x^3)$ **d** $4x^{-2}(3x^5 + 2x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$ **f** $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$ **g** $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$ **h** $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

Express as individual terms where possible

a $\frac{x^3 + 2x}{x}$ **b** $\frac{4t^5 - 6t^3}{2t^2}$ **c** $\frac{x^2}{3x^3 - x}$ **d** $\frac{x^{\frac{3}{2}} - 3x}{x^{\frac{1}{2}}}$

e $\frac{3y}{y^2(y^3 - 6)}$ **f** $\frac{p + p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$ **g** $\frac{8w - 2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$ **h** $\frac{x^{\frac{1}{2}} + 3}{x^3 - x}$

Section 4: Using the laws of indices - **Optional**

Video: https://youtu.be/Q_F74bv-bRk

Examples	**Your turn**
At A level:	
If $y = 2^x$, express in terms of y:	If $y = 3^x$, express in terms of y:
2^{2x}	3^{3x}
4^x	9^x
2^{2x+1}	3^{3x-1}
Solve $x^{\frac{2}{3}} = 64$	Solve $x^{\frac{3}{2}} = 64$

Express in the form 2^k

a 8^2 **b** $(\frac{1}{4})^{-2}$ **c** $(\frac{1}{2})^{\frac{1}{3}}$ **d** $16^{-\frac{1}{6}}$ **e** $8^{\frac{2}{3}}$ **f** $(\frac{1}{32})^{-3}$

Given that $y = 2^x$, express each of the following in terms of y.

a 2^{x+1} **b** 2^{x-2} **c** 2^{2x} **d** 8^x **e** 2^{4x+3} **f** $(\frac{1}{2})^{x-3}$

Solve each equation.

a $x^{\frac{1}{2}} = 6$ **b** $x^{\frac{1}{3}} = 5$ **c** $x^{-\frac{1}{2}} = 2$ **d** $x^{-\frac{1}{4}} = \frac{1}{3}$

e $x^{\frac{3}{2}} = 8$ **f** $x^{\frac{2}{3}} = 16$ **g** $x^{\frac{4}{3}} = 81$ **h** $x^{-\frac{3}{2}} = 27$

Fractions

Section 1: Multiplying and simplifying - **Compulsory**

Video: https://youtu.be/8q_2Ydq8st0

Examples	**Your turn**
At GCSE:	
Simplify: $\frac{10}{15}$ $55 \times \frac{12}{33}$	Simplify: $\frac{12}{18}$ $14 \times \frac{15}{35}$
At A level:	
Simplify, where possible:  $\frac{4x + 8y}{4}$ $\frac{4x \times 8y}{4}$ $\frac{4x + y}{4}$ $\frac{x^2 - 1}{3x + 3}$ $\frac{3x^2}{9x - 9} \times \frac{4x - 4}{2x}$	Simplify, where possible: $\frac{3x + 9y}{3}$ $\frac{3x \times 9y}{3}$ $\frac{3x + y}{3}$ $\frac{2x - 6}{x^2 - x - 6}$ $\frac{2x + 8}{x} \times \frac{3x^2}{3x + 12}$

Simplify, where possible:

a $\dfrac{4+6x}{2}$ **b** $\dfrac{4\times 6x}{2}$ **c** $\dfrac{3x+xy}{x}$ **d** $\dfrac{3x\times xy}{x}$

e $\dfrac{4x+10y}{8x+6y}$ **f** $\dfrac{3x-6y}{9x-3y}$ **g** $\dfrac{4x+9y}{2x+3y}$ **h** $\dfrac{5xy+6y^2}{10x+12y}$

i $\dfrac{3x^2+4y^2}{6x^2-8y^2}$ **j** $\dfrac{x-3}{3-x}$ **k** $\dfrac{3y+3}{y^2+7y+6}$ **l** $\dfrac{x^2-25}{x^2-7x+10}$

m $\dfrac{n^2+2n}{n^2+6n+8}\times\dfrac{n+4}{n^2}$ **n** $\dfrac{2x+8}{x}\times\dfrac{3x^2}{3x+12}$ **o** $\dfrac{x^2+7x+12}{x^2+2x-3}\times\dfrac{x+1}{x+4}$

Section 2: Adding and subtracting - **Optional**

Video: https://youtu.be/e1K5a9nvr64

Examples	**Your turn**
At GCSE:	
$\frac{3}{5}-\frac{1}{7}$	$\frac{1}{4}+\frac{5}{9}$
$\frac{1}{3}+\frac{1}{9}$	$\frac{1}{4}-\frac{3}{8}$
$2-\frac{1}{2}+\frac{1}{4}$ 	$1-\frac{2}{3}+\frac{1}{6}$
At A level:	
$\frac{2}{x+3}-\frac{4}{x-1}$	$\frac{3}{x-2}+\frac{2}{x+1}$
$\frac{2}{x+3}-\frac{4}{(x+3)^2}$	$\frac{3}{x-2}+\frac{2}{(x-2)^2}$
$2-\frac{5}{(x+2)(x-1)}+\frac{3}{(x+2)(x-3)}$	$1-\frac{4}{(x-4)(x-3)}+\frac{2}{(x-3)(x+4)}$

Express as a single fraction in its simplest form

a $\frac{2}{y}+\frac{7}{y+4}$ **b** $\frac{2x}{x-5}-\frac{1}{x+3}$ **c** $\frac{7}{x(x+2)}-\frac{3x}{x+2}$

d $\frac{x}{(x-3)(x-1)}+\frac{5}{2(x-1)}$ **e** $\frac{2}{q^2+3q}+\frac{5q}{4q+12}$ **f** $\frac{4}{3x-3}+\frac{x+2}{x^2-x}$

g $\frac{4}{x+5}+\frac{x}{x^2+8x+15}$ **h** $\frac{6x}{x^2-4}-\frac{3}{x+2}$ **i** $\frac{5t+12}{2t^2+7t+3}-\frac{4}{2t+1}$

Express as a single fraction in its simplest form

a $\frac{5}{x^2-1}-\frac{1}{2x+2}$ **b** $\frac{3x}{x^2-4}-\frac{4}{2x^2+3x-2}$

c $\frac{4}{x^2+2x-3}+\frac{1}{x^2-3x+2}$ **d** $\frac{x+1}{x^2-25}+\frac{2}{x^2+5x}$

Section 3: Dividing - **Compulsory**

Video: https://youtu.be/SSrwYr0VZ50

Examples	**Your turn**
At GCSE:	
$\dfrac{\left(\frac{1}{2}\right)}{3}$ 	$\dfrac{\left(\frac{1}{4}\right)}{5}$
$\dfrac{1}{\left(\frac{2}{3}\right)}$	$\dfrac{1}{\left(\frac{4}{5}\right)}$
$\dfrac{\left(\frac{1}{2}\right)}{\left(\frac{1}{3}\right)}$	$\dfrac{\left(\frac{1}{4}\right)}{\left(\frac{1}{5}\right)}$
At A level:	
$\dfrac{\left(\frac{3y^2}{4x}\right)}{y}$	$\dfrac{\left(\frac{2x^3}{3y}\right)}{x^2}$
$\dfrac{y}{\left(\frac{3y^2}{4x}\right)}$	$\dfrac{x^2}{\left(\frac{2x^3}{3y}\right)}$
$\dfrac{\frac{1}{x-1}+1}{\frac{1}{x-1}-1}$	$\dfrac{1+\frac{1}{x}}{2-\frac{3}{x}}$

Express as a single fraction

a $\dfrac{\left(\frac{5x^3}{4y}\right)}{x}$ **b** $\dfrac{5x^3}{\left(\frac{4y}{x}\right)}$ **c** $\dfrac{\left(\frac{6x^2y^3}{5z}\right)}{2xy}$ **d** $\dfrac{\left(\frac{5a^2}{6x^3z^2}\right)}{2y}$

e $\dfrac{\left(\frac{2\pi x}{ab}\right)}{\frac{1}{3}\pi r^3}$ **f** $\dfrac{\frac{2}{x}+1}{\frac{3}{x}-1}$ **g** $\dfrac{\frac{1}{x+1}+2}{\frac{1}{x+1}+1}$ **h** $\dfrac{\frac{x^2}{\sqrt{x^2+1}}-\sqrt{x^2+1}}{x^2}$

Answers

Algebra Basics

Exercise 1

$a(x+y)$	
$ax+y$	$ax+ay$
$a(x\times y)$	
$ax\times y$	$ax\times ay$
$(x+y)^2$	
x^2+y^2	x^2+y^2+2xy
$(3x)^2$	
$9x^2$	$3x^2$
$(x+y)^3$	
$x^3+y^3+3x^2y+3xy^2$	x^3+y^3
$\left(\sqrt{x+y}\right)^2$	
$x+y$	$x+y+2\sqrt{x}\sqrt{y}$
$x+y$	
$\sqrt{x^2+y^2}$	$\sqrt{x^2}+\sqrt{y^2}$
$\sqrt{4x}$	
$4\sqrt{x}$	$2\sqrt{x}$
$9(x+y)^2$	
$(9x+9y)^2$	$(3x+3y)^2$
$\sqrt{9x+9y}$	
$9\sqrt{x+y}$	$3\sqrt{x+y}$
$\left(\frac{2}{x}\right)^3$	
$\frac{2}{x^3}$	$\frac{8}{x^3}$
$\frac{\sqrt{20}}{2}$	
$\sqrt{5}$	$\sqrt{10}$
2^{x+y}	
2^x+2^y	$2^x\times 2^y$
6^x	
3×2^x	$3^x\times 2^x$
$\sin(x+y)$	
$\sin(x)+\sin(y)$	$\sin(x)\cos(y)+\sin(y)\cos(x)$
$\sin(2x)$	
$2\sin(x)$	$2\sin(x)\cos(x)$

Exercise 2

1. $x^4-5x^3+14x^2+6$
2. $4(x+3)^2-4(x+1)^2-7(x+3)$
3. $3\sin(x)-6\sin(2x)+3\cos(2x)$
4. $-3^x+4\times 9^x$
5. x^2

Exercise 3

a $= x(x^2+5x-6)+(x^2+5x-6)$
$= x^3+5x^2-6x+x^2+5x-6$
$= x^3+6x^2-x-6$

b $= 2x(x^2-3x+7)-5(x^2-3x+7)$
$= 2x^3-6x^2+14x-5x^2+15x-35$
$= 2x^3-11x^2+29x-35$

c $= 4(2+5x-x^2)-7x(2+5x-x^2)$
$= 8+20x-4x^2-14x-35x^2+7x^3$
$= 7x^3-39x^2+6x+8$

d $= (3x-2)(3x-2)^2=(3x-2)(9x^2-12x+4)$
$= 3x(9x^2-12x+4)-2(9x^2-12x+4)$
$= 27x^3-36x^2+12x-18x^2+24x-8$
$= 27x^3-54x^2+36x-8$

e $= x^2(2x^2-x+9)+3(2x^2-x+9)$
$= 2x^4-x^3+9x^2+6x^2-3x+27$
$= 2x^4-x^3+15x^2-3x+27$

f $= 4x(x^4-3x^2+5x+2)-(x^4-3x^2+5x+2)$
$= 4x^5-12x^3+20x^2+8x-x^4+3x^2-5x-2$
$= 4x^5-x^4-12x^3+23x^2+3x-2$

g $= x^2(x^2+3x+1)+2x(x^2+3x+1)+5(x^2+3x+1)$
$= x^4+3x^3+x^2+2x^3+6x^2+2x+5x^2+15x+5$
$= x^4+5x^3+12x^2+17x+5$

h $= x^2(2x^2-x+4)+x(2x^2-x+4)-3(2x^2-x+4)$
$= 2x^4-x^3+4x^2+2x^3-x^2+4x-6x^2+3x-12$
$= 2x^4+x^3-3x^2+7x-12$

i $= 3x^2(2x^2-4x-8)-5x(2x^2-4x-8)+2(2x^2-4x-8)$
$= 6x^4-12x^3-24x^2-10x^3+20x^2+40x+4x^2-8x-16$
$= 6x^4-22x^3+32x-16$

j $= x^2(x^2+2x-6)+2x(x^2+2x-6)-6(x^2+2x-6)$
$= x^4+2x^3-6x^2+2x^3+4x^2-12x-6x^2-12x+36$
$= x^4+4x^3-8x^2-24x+36$

k $= x^3(2x^4+x^2+3)+4x^2(2x^4+x^2+3)+(2x^4+x^2+3)$
$= 2x^7+x^5+3x^3+8x^6+4x^4+12x^2+2x^4+x^2+3$
$= 2x^7+8x^6+x^5+6x^4+3x^3+13x^2+3$

l $= 6(3+x^2-x^3+2x^4)-2x(3+x^2-x^3+2x^4)+x^3(3+x^2-x^3+2x^4)$
$= 18+6x^2-6x^3+12x^4-6x-2x^3+2x^4-4x^5+3x^3+x^5-x^6+2x^7$
$= 2x^7-x^6-3x^5+14x^4-5x^3+6x^2-6x+18$

Exercise 4

1. $(x+1)(x-3)[(x+1)+(x-3)]=0$
   $(x+1)(x-3)(2x-2)=0$
   $x=-1,3,1$
2. $(2x+1)^2[(2x+1)-(x+4)]=0$
   $(2x+1)^2(x-3)=0$
   $x=-\frac{1}{2},3$
3. $(3x-2)^3(2x+3)^3[(3x-2)+1]=0$
   $(3x-2)^3(2x+3)^3(3x-1)=0$
   $x=\frac{2}{3},-\frac{3}{2},\frac{1}{3}$
4. $(1-x)^4(2-5x)^6[(2-5x)-(1-x)]=0$
   $(1-x)^4(2-5x)^6(1-4x)=0$
   $x=1,\frac{2}{5},\frac{1}{4}$

a $x = \frac{3}{2}, -1$ **b** $x = \frac{3}{2}, -1$ **c** $x = 2, -3$

d $x = 1,3, -3$ **e** $x = 0,1$ **f** $x = 1,0,180,360,540, \ldots.$

Indices

Exercise 1

a $= x^2$ **b** $= y^{-6}$ **c** $= 3p^{-4}$ **d** $= 8x^{-12}$

e $= y^{\frac{5}{12}}$ **f** $= 8b^{\frac{2}{3}+\frac{1}{4}} = 8b^{\frac{11}{12}}$ **g** $= x^{\frac{3}{5}-\frac{1}{3}} = x^{\frac{4}{15}}$ **h** $= a^{\frac{1}{2}-\frac{4}{3}} = a^{-\frac{5}{6}}$

i $= p^{\frac{1}{4}-(-\frac{1}{5})} = p^{\frac{9}{20}}$ **j** $= 9x^{\frac{4}{5}}$ **k** $= y^{1+\frac{5}{6}-\frac{3}{2}} = y^{\frac{1}{3}}$ **l** $= \frac{1}{3}t$

a $= x^{\frac{1}{2}}$ **b** $= x^{-\frac{1}{3}}$ **c** $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$ **d** $= \frac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$

e $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$ **f** $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$ **g** $= (x^{\frac{1}{5}})^5 = x^{\frac{5}{5}}$ **h** $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$

Exercise 2

a $4x^{-\frac{1}{2}}$ **b** $\frac{1}{2}x^{-1}$ **c** $\frac{3}{4}x^{-3}$ **d** $\frac{1}{9}x^{-2}$ **e** $\frac{2}{5}x^{-\frac{1}{3}}$ **f** $\frac{1}{3}x^{-\frac{3}{2}}$

Exercise 3

a $= x^3 - 1$ **b** $= 2x^2 + 6x^3$ **c** $= 3 - x^2$ **d** $= 12x^3 + 8x$

e $= 3x^3 + 2x$ **f** $= 3 - 3x^2$ **g** $= 5x^{\frac{1}{2}} + x^2$ **h** $= 3x^2 - x^{-1}$

a $x^2 + 2$ **b** $2t^3 - 3t$ **c** Not possible **d** $x - 3x^{\frac{1}{2}}$

e Not possible **f** $p^{\frac{1}{4}} + p^{\frac{3}{4}}$ **g** $2w^{\frac{3}{2}} - \frac{1}{2}w$ **h** Not possible

Exercise 4

a $= (2^3)^2 = 2^6$ **b** $= (2^{-2})^{-2} = 2^4$ **c** $= (2^{-1})^{\frac{1}{3}} = 2^{-\frac{1}{3}}$

d $= (2^4)^{-\frac{1}{6}} = 2^{-\frac{2}{3}}$ **e** $= (2^3)^{\frac{2}{5}} = 2^{\frac{6}{5}}$ **f** $= (2^{-5})^{-3} = 2^{15}$

a $= 2 \times 2^x = 2y$ **b** $= 2^{-2} \times 2^x = \frac{1}{4}y$ **c** $= (2^x)^2 = y^2$

d $= (2^3)^x = 2^{3x} = (2^x)^3 = y^3$ **e** $= 2^3 \times 2^{4x} = 8y^4$ **f** $= (2^{-1})^{x-3} = 2^3 \times 2^{-x} = \frac{8}{y}$

a $x = 6^2 = 36$ **b** $x = 5^3 = 125$ **c** $x^{\frac{1}{2}} = \frac{1}{2}$, $x = (\frac{1}{2})^2 = \frac{1}{4}$ **d** $x^{\frac{1}{4}} = 3$, $x = 3^4 = 81$

e $x^{\frac{1}{2}} = \sqrt[3]{8} = 2$, $x = 2^2 = 4$ **f** $x^{\frac{1}{3}} = \pm\sqrt{16} = \pm 4$, $x = (\pm 4)^3 = \pm 64$ **g** $x^{\frac{1}{3}} = \pm\sqrt[4]{81} = \pm 3$, $x = (\pm 3)^3 = \pm 27$ **h** $x^{\frac{3}{2}} = \frac{1}{27}$, $x^{\frac{1}{3}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$, $x = (\frac{1}{3})^2 = \frac{1}{9}$

Fractions

Exercise 1

a $2+3x$ **b** $12x$ **c** $3+y$ **d** $3xy$

e $\dfrac{2x+5y}{4x+3y}$ **f** $\dfrac{x-2y}{3x-y}$ **g** Not possible **h** $\dfrac{y}{2}$

i Not possible **j** $\dfrac{2}{3}$ **k** $= \dfrac{3(y+1)}{(y+6)(y+1)} = \dfrac{3}{y+6}$ **l** $= \dfrac{(x+5)(x-5)}{(x-2)(x-5)} = \dfrac{x+5}{x-2}$

m $= \dfrac{n(n+2)}{(n+4)(n+2)} \times \dfrac{n+4}{n^2} = \dfrac{1}{n}$ **n** $2x$ **o** $\dfrac{x+1}{x-1}$

Exercise 2

a $= \dfrac{2(y+4)+7y}{y(y+4)}$

$= \dfrac{9y+8}{y(y+4)}$

b $= \dfrac{2x(x+3)-(x-5)}{(x-5)(x+3)}$

$= \dfrac{2x^2+5x+5}{(x-5)(x+3)}$

c $= \dfrac{7-3x^2}{x(x+2)}$

d $= \dfrac{2x+5(x-3)}{2(x-3)(x-1)}$

$= \dfrac{7x-15}{2(x-3)(x-1)}$

e $= \dfrac{2}{q(q+3)} + \dfrac{5q}{4(q+3)}$

$= \dfrac{8+5q^2}{4q(q+3)}$

f $= \dfrac{4}{3(x-1)} + \dfrac{x+2}{x(x-1)}$

$= \dfrac{4x+3(x+2)}{3x(x-1)} = \dfrac{7x+6}{3x(x-1)}$

g $= \dfrac{4}{x+5} + \dfrac{x}{(x+3)(x+5)}$

$= \dfrac{4(x+3)+x}{(x+3)(x+5)}$

$= \dfrac{5x+12}{(x+3)(x+5)}$

h $= \dfrac{6x}{(x+2)(x-2)} - \dfrac{3}{x+2}$

$= \dfrac{6x-3(x-2)}{(x+2)(x-2)}$

$= \dfrac{3x+6}{(x+2)(x-2)}$

$= \dfrac{3(x+2)}{(x+2)(x-2)} = \dfrac{3}{x-2}$

i $= \dfrac{5t+12}{(2t+1)(t+3)} - \dfrac{4}{2t+1}$

$= \dfrac{5t+12-4(t+3)}{(2t+1)(t+3)}$

$= \dfrac{t}{(2t+1)(t+3)}$

a $= \dfrac{5}{(x+1)(x-1)} - \dfrac{1}{2(x+1)}$

$= \dfrac{10-(x-1)}{2(x+1)(x-1)}$

$= \dfrac{11-x}{2(x+1)(x-1)}$

b $= \dfrac{3x}{(x+2)(x-2)} - \dfrac{4}{(2x-1)(x+2)}$

$= \dfrac{3x(2x-1)-4(x-2)}{(x+2)(x-2)(2x-1)}$

$= \dfrac{6x^2-7x+8}{(x+2)(x-2)(2x-1)}$

c $= \dfrac{4}{(x+3)(x-1)} + \dfrac{1}{(x-1)(x-2)}$

$= \dfrac{4(x-2)+(x+3)}{(x+3)(x-1)(x-2)}$

$= \dfrac{5x-5}{(x+3)(x-1)(x-2)}$

$= \dfrac{5}{(x+3)(x-2)}$

d $= \dfrac{x+1}{(x+5)(x-5)} + \dfrac{2}{x(x+5)}$

$= \dfrac{x(x+1)+2(x-5)}{x(x+5)(x-5)}$

$= \dfrac{x^2+3x-10}{x(x+5)(x-5)}$

$= \dfrac{(x+5)(x-2)}{x(x+5)(x-5)}$

$= \dfrac{x-2}{x(x-5)}$

Exercise 3

a $\frac{5x^2}{4y}$ **b** $\frac{5x^4}{4y}$ **c** $\frac{3xy^2}{5z}$ **d** $\frac{5a^2}{12x^3yz^2}$

e $\frac{6x}{abr^3}$ **f** $\frac{2+x}{3-x}$ **g** $\frac{2x+3}{x+2}$ **h** $\frac{-1}{x^2\sqrt{x^2+1}}$