Summer Independent Learning:

A level Maths for Further Maths

Year 11 into Year 12

This is the SIL for both Maths **and** Further Maths (please do not do the regular Maths SIL). This means there is (roughly) twice as much as for your other subjects, as it is for two sets of lessons.

Welcome to A level maths!

These activities have been designed to help you prepare for the initial assessment in September, which will indicate how well-prepared you are for A level study. It is based on key GCSE Higher content which forms the basis for our first few A level topics.

If you have access to a printer please print pages 2 and 3 of this document and fill them in as you complete the tasks. If not, please make a copy of each table and fill it in.

You do not need to print the other pages and should complete all questions **on separate paper** so you have enough room to show full detailed methods. Don't be tempted to just write the answers!

Please bring your **completed tables** and **all the work you have done** to your first lesson.

Section 1: Preparation Work

1. Complete the practice questions for each topic. The videos are provided for additional support.
2. Mark and correct your work in a different colour.
3. Red/amber/green rate your confidence with this topic.
4. Make a note of any questions you would like to ask in September.

Topic	***Done and marked? ✓***	***RAG Rating***	**Comments/questions**
B1 Indices			
B2 Surds			
B3 Quadratics			
B4 Simultaneous equations			
B5 Inequalities			
Rearranging equations			
E1 Triangle geometry			
E7 Trig equations			
E3 Exact trig values			

Section 2: Practice Papers

1. Attempt Practice Test 1 without referring to any notes or videos.
2. Mark and correct your work in a different colour.
3. Red/amber/green rate how well you did with this topic.
4. Go back to the Task 1 questions/videos and do some more practice for any red/amber rated topics.

Topic	***RAG Rating***	**Comments/questions**
B1 Indices		
B2 Surds		
B3 Quadratics		
B4 Simultaneous equations		
B5 Inequalities		
E1 Triangle geometry		
Rearranging equations		
E7 Trig equations		
E3 Exact trig values		

5. Repeat steps 1-4 for Practice Test 2.

Topic	***RAG Rating***	**Comments/questions**
B1 Indices		
B2 Surds		
B3 Quadratics		
B4 Simultaneous equations		
B5 Inequalities		
E1 Triangle geometry		
Rearranging equations		
E7 Trig equations		
E3 Exact trig values		

B1 Indices

Videos:

https://youtu.be/1lThXgU08S0

https://youtu.be/v5bn4HZrmQs

https://youtu.be/W0h4rHj88ys

Question 1

Express in the form x^k

a $\sqrt{x}$ **b** $\frac{1}{\sqrt[3]{x}}$ **c** $x^2 \times \sqrt{x}$ **d** $\frac{\sqrt[4]{x}}{x}$

e $\sqrt{x^3}$ **f** $\sqrt{x} \times \sqrt[3]{x}$ **g** $(\sqrt{x})^5$ **h** $\sqrt[3]{x^2} \times (\sqrt{x})^3$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$ **j** $(3x^{\frac{2}{5}})^2$ **k** $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$ **l** $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\frac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$ **n** $\frac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$ **o** $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{1}{4}}}$ **p** $\frac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

Question 2

Express each of the following in the form 3^y, where y is a function of x.

a 9^x **b** 81^{x+1} **c** $27^{\frac{x}{3}}$ **d** $(\frac{1}{3})^x$ **e** 9^{2x-1} **f** $(\frac{1}{27})^{x+2}$

Question 3

Simplify in to one or more terms of the form ax^n (where a and n are constants to be found, and not all questions use x).

a $\frac{x^3+2x}{x}$ **b** $\frac{4t^5-6t^3}{2t^2}$ **c** $\frac{x^{\frac{3}{2}}-3x}{x^{\frac{1}{2}}}$ **d** $\frac{y^2(y^3-6)}{3y}$

e $\frac{p+p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$ **f** $\frac{8w-2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$ **g** $\frac{x+1}{x^{\frac{1}{2}}+x^{-\frac{1}{2}}}$ **h** $\frac{2t^3-4t}{t^{\frac{3}{2}}-2t^{-\frac{1}{2}}}$

Exam style question

Solve the equation

$$25^x = 5^{4x+1}.$$

B1 Indices – Answers

Question 1

a $= x^{\frac{1}{2}}$ **b** $= x^{-\frac{1}{3}}$ **c** $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$ **d** $= \frac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$

e $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$ **f** $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$ **g** $= (x^{\frac{1}{2}})^5 = x^{\frac{5}{2}}$ **h** $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$

i $= p^{\frac{1}{4}-(-\frac{1}{5})} = p^{\frac{9}{20}}$ **j** $= 9x^{\frac{4}{5}}$ **k** $= y^{1+\frac{5}{6}-\frac{3}{2}} = y^{\frac{1}{3}}$ **l** $= \frac{1}{3}t$

m $= b^{2+\frac{1}{4}-\frac{1}{2}} = b^{\frac{7}{4}}$ **n** $= y^{\frac{1}{2}+\frac{1}{3}-1} = y^{-\frac{1}{6}}$ **o** $= 2x^{\frac{2}{3}+(-\frac{1}{6})-\frac{3}{4}} = 2x^{-\frac{1}{4}}$ **p** $= \frac{1}{4}a^{1+\frac{3}{4}-(-\frac{1}{2})} = \frac{1}{4}a^{\frac{9}{4}}$

Question 2

a $= (3^2)^x = 3^{2x}$ **b** $= (3^4)^{x+1} = 3^{4x+4}$ **c** $= (3^3)^{\frac{x}{4}} = 3^{\frac{3}{4}x}$

d $= (3^{-1})^x = 3^{-x}$ **e** $= (3^2)^{2x-1} = 3^{4x-2}$ **f** $= (3^{-3})^{x+2} = 3^{-3x-6}$

Question 3

a $= x^2 + 2$ **b** $= 2t^3 - 3t$ **c** $= x - 3x^{\frac{1}{2}}$ **d** $= \frac{y^5 - 6y^2}{3y}$

$= \frac{1}{3}y^4 - 2y$

e $= p^{\frac{1}{4}} + p^{\frac{3}{4}}$ **f** $= 2w^{\frac{3}{2}} - \frac{1}{2}w$ **g** $= \frac{x^{\frac{1}{2}}(x+1)}{x+1}$

$= x^{\frac{1}{2}}$

h $= \frac{t^{\frac{1}{2}} \times 2t(t^2-2)}{t^2-2}$

$= 2t^{\frac{3}{2}}$

Exam style question

$25^x = (5^2)^x = 5^{4x+1}$

$5^{2x} = 5^{4x+1}$

$2x = 4x + 1$

$x = -\frac{1}{2}$

B2 Surds

Video: https://youtu.be/jHelde32YtI

Question 1

Simplify

a $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$

Question 2

Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ **b** $4 - \sqrt{3} - 2(1 - \sqrt{3})$ **c** $(1 + \sqrt{3})(2 + \sqrt{3})$

Question 3

Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{2}{\sqrt{3}}$ **c** $\frac{1}{\sqrt{8}}$ **d** $\frac{14}{\sqrt{7}}$

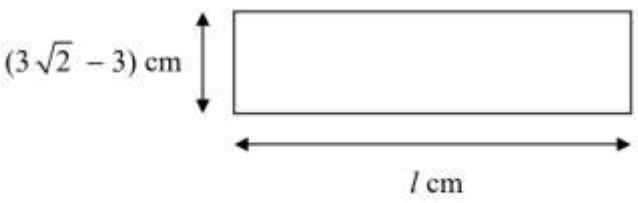
Question 4

Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2}+1}$ **b** $\frac{4}{\sqrt{3}-1}$ **c** $\frac{1}{\sqrt{6}-2}$ **d** $\frac{3}{2+\sqrt{3}}$

Exam style questions

(a)



The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm.

Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

(b)

Given that n is a positive integer, express

$$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3}$$

as a single fraction not involving surds.

(c)

Solve the equation

$$3x = \sqrt{5}\,(x+2),$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.

B2 Surds – Answers

Question 1

a $= 3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$ **b** $= 4\sqrt{3}-3\sqrt{3}=\sqrt{3}$ **c** $= 4\sqrt{2}+6\sqrt{2}=10\sqrt{2}$

Question 2

a $= 3+2\sqrt{3}$

b $= 4-\sqrt{3}-2+2\sqrt{3}$
$= 2+\sqrt{3}$

c $= 2+\sqrt{3}+2\sqrt{3}+3$
$= 5+3\sqrt{3}$

Question 3

a $= \frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$ **b** $= \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$ **c** $= \frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}\sqrt{2}$

d $= \frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = 2\sqrt{7}$

Question 4

a $= \frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$

b $= \frac{4}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1)$

c $= \frac{1}{\sqrt{6}-2}\times\frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{\sqrt{6}+2}{6-4} = \frac{1}{2}(\sqrt{6}+2)$ or $\frac{1}{2}\sqrt{6}+1$

d $= \frac{3}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3})$

Exam style questions

(a)

$$l = \frac{6}{3\sqrt{2}-3} = \frac{6}{3\sqrt{2}-3}\times\frac{3\sqrt{2}+3}{3\sqrt{2}+3} = \frac{6(3\sqrt{2}+3)}{18-9}$$

$$l = \frac{18(\sqrt{2}+1)}{9} = 2\sqrt{2}+2$$

(b)

$$\frac{7}{3+5\sqrt{n}}-\frac{7}{5\sqrt{n}-3}$$

$$=\frac{7\left(5\sqrt{n}-3\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}-\frac{7\left(3+5\sqrt{n}\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}$$

$$=\frac{35\sqrt{n}-21-\left(21+35\sqrt{n}\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}$$

$$=-\frac{42}{25n-9}$$

(c)

$$3x=\sqrt{5}\,x+2\sqrt{5}$$

$$x(3-\sqrt{5})=2\sqrt{5}$$

$$x=\frac{2\sqrt{5}}{3-\sqrt{5}}=\frac{2\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{2\sqrt{5}(3+\sqrt{5})}{9-5}$$

$$x=\frac{6\sqrt{5}+10}{4}=\frac{5}{2}+\frac{3}{2}\sqrt{5}$$

B3 Quadratics

Videos:

https://youtu.be/Pziws8ojnlk

https://youtu.be/sn_joGVj15w

https://youtu.be/kk7p6hjn7hQ

https://youtu.be/toIqbX_NXHo

Question 1

Factorise:

(a)	$x^2 - 3x + 2$	(b)	$x^2 + 5x + 6$	(c)	$x^2 - 9$
(d)	$x^2 - 10x + 25$	(e)	$2x^2 - 3x + 1$	(f)	$5x^2 - 17x + 6$
(g)	$16 - 9x^2$	(h)	$x^4 + 4x^2 + 3$	(i)	$x^5 - 4x^3 + 4x$

Question 2

Hence, sketch (showing the coordinates of any points of intersections with coordinate axes):

(a)	$y = x^2 - 3x + 2$	(b)	$y = x^2 + 5x + 6$	(c)	$y = x^2 - 9$
(d)	$y = x^2 - 10x + 25$	(e)	$y = 2x^2 - 3x + 1$	(f)	$y = 5x^2 - 17x + 6$

Question 3

Complete the square, leaving in the form $(x + a)^2 + b$ or $a(x + b)^2 + c$, where appropriate

(a)	$x^2 - 4x + 3$	(b)	$x^2 + 8x + 30$	(c)	$x^2 - 5x + 4$
(d)	$x^2 + 3x + 3$	(e)	$4x^2 + 8x + 3$	(f)	$8 + 2x - x^2$

Question 4

Hence, sketch (showing the coordinates of turning point, and y intercept):

(a)	$y = x^2 - 4x + 3$	(b)	$y = x^2 + 8x + 30$	(c)	$y = x^2 - 5x + 4$
(d)	$y = x^2 + 3x + 3$	(e)	$y = 4x^2 + 8x + 3$	(f)	$y = 8 + 2x - x^2$

Exam style questions

(i)

a Express $x^2 - 4\sqrt{2}\,x + 5$ in the form $a(x + b)^2 + c$.

b Write down an equation of the line of symmetry of the curve $y = x^2 + 4\sqrt{2}\,x + 5$.

(ii)

$$\mathrm{f}(x) \equiv x^2 + 2kx - 3.$$

By completing the square, find the roots of the equation $\mathrm{f}(x) = 0$ in terms of the constant k.

(iii)

By completing the square, show that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

B3 Quadratics – Answers

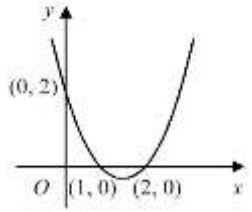
Question 1

(a)	$(x-1)(x-2)$	(b)	$(x+3)(x+2)$	€	$(x+3)(x-3)$
(d)	$(x-5)^2$	€	$(2x-1)(x-1)$	(f)	$(5x-2)(x-3)$
(g)	$(4+3x)(4-3x)$	(h)	$(x^2+3)(x^2+1)$	(i)	$x(x^4-4x^2+4)$ $x(x^2-2)^2$

Question 2

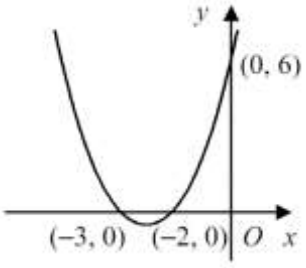
(a)

$x^2 - 3x + 2 = 0$
$(x-1)(x-2) = 0$
$x = 1$ or 2



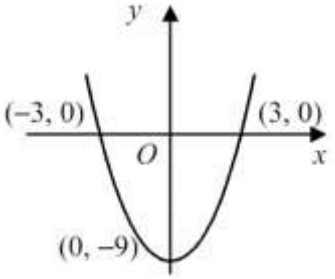
(b)

$x^2 + 5x + 6 = 0$
$(x+3)(x+2) = 0$
$x = -3$ or -2



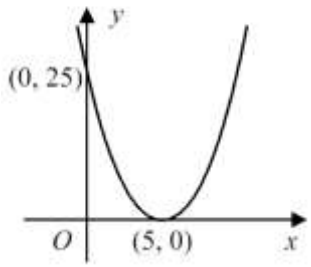
(c)

$x^2 - 9 = 0$
$(x+3)(x-3) = 0$
$x = -3$ or 3



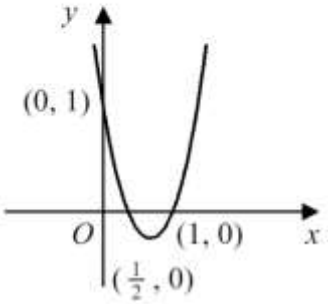
(d)

$x^2 - 10x + 25 = 0$
$(x-5)^2 = 0$
$x = 5$



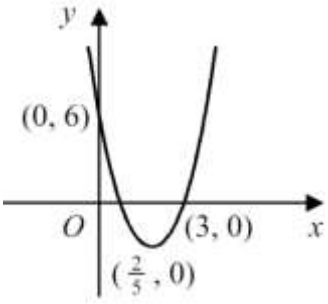
(e)

$2x^2 - 3x + 1 = 0$
$(2x-1)(x-1) = 0$
$x = \frac{1}{2}$ or 1



(f)

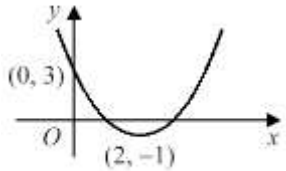
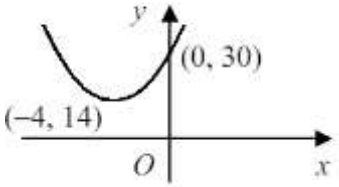
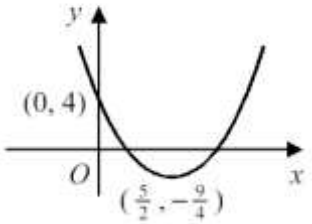
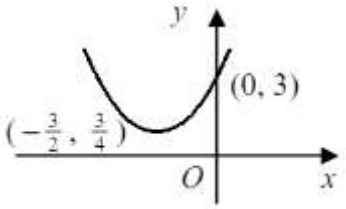
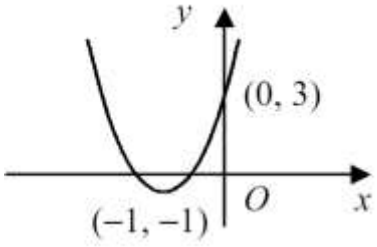
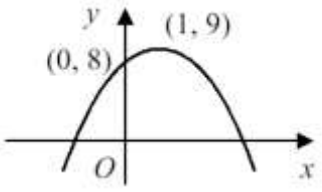
$5x^2 - 17x + 6 = 0$
$(5x-2)(x-3) = 0$
$x = \frac{2}{5}$ or 3



Question 3

(a)	$y=(x-2)^2-4+3$ $y=(x-2)^2-1$	(b)	$y=(x+4)^2-16+30$ $y=(x+4)^2+14$	(c)	$y=(x-\frac{5}{2})^2-\frac{25}{4}+4$ $y=(x-\frac{5}{2})^2-\frac{9}{4}$
(d)	$y=(x+\frac{3}{2})^2-\frac{9}{4}+3$ $y=(x+\frac{3}{2})^2+\frac{3}{4}$	(e)	$y=4[x^2+2x]+3$ $y=4[(x+1)^2-1]+3$ $y=4(x+1)^2-1$	(f)	$y=-[x^2-2x]+8$ $y=-[(x-1)^2-1]+8$ $y=-(x-1)^2+9$

Question 4

(a)	$y=(x-2)^2-4+3$ $y=(x-2)^2-1$ minimum $(2,-1)$ 	(b)	$y=(x+4)^2-16+30$ $y=(x+4)^2+14$ minimum $(-4, 14)$ 	(c)	$y=(x-\frac{5}{2})^2-\frac{25}{4}+4$ $y=(x-\frac{5}{2})^2-\frac{9}{4}$ minimum $(\frac{5}{2}, -\frac{9}{4})$ 
(d)	$y=(x+\frac{3}{2})^2-\frac{9}{4}+3$ $y=(x+\frac{3}{2})^2+\frac{3}{4}$ minimum $(-\frac{3}{2}, \frac{3}{4})$ 	(e)	$y=4[x^2+2x]+3$ $y=4[(x+1)^2-1]+3$ $y=4(x+1)^2-1$ minimum $(-1, -1)$ 	(f)	$y=-[x^2-2x]+8$ $y=-[(x-1)^2-1]+8$ $y=-(x-1)^2+9$ maximum $(1, 9)$ 

Exam style questions

(i)

a $= (x - 2\sqrt{2})^2 - 8 + 5$

$= (x - 2\sqrt{2})^2 - 3$

b $x = 2\sqrt{2}$

(ii)

$x^2 + 2kx - 3 = 0$

$(x + k)^2 - k^2 - 3 = 0$

$(x + k)^2 = k^2 + 3$

$x + k = \pm\sqrt{k^2 + 3}$

$x = -k \pm \sqrt{k^2 + 3}$

(iii)

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

B4 Simultaneous equations

Video: https://youtu.be/4SRtwS5unwE

Solve these pairs of simultaneous equations:

(a)	$y = 2x + 6$ $y = 3 - 4x$	(b)	$3x + 3y + 4 = 0$ $5x - 2y - 5 = 0$	€	$x^2 - y + 3 = 0$ $x - y + 5 = 0$
(d)	$2x^2 - y - 8x = 0$ $x + y + 3 = 0$	(e)	$x^2 - 4y - y^2 = 0$ $x - 2y = 0$	(f)	$xy = 6$ $x - y = 5$
(g)	$\frac{3}{x} - 2y + 4 = 0$ $4x + y - 7 = 0$	(h)	$y = 2^x$ $4^x + y = 72$	(i)	$3^{x-1} = 9^{2y}$ $8^{x-2} = 4^{1+y}$

B4 Simultaneous equations – Answers

(a)

$2x + 6 = 3 - 4x$

$x = -\frac{1}{2}$

$\therefore\ x = -\frac{1}{2}, y = 5$

(b)

$6x + 6y + 8 = 0$

$15x - 6y - 15 = 0$

adding

$21x - 7 = 0$

$x = \frac{1}{3}$

$\therefore\ x = \frac{1}{3}, y = -\frac{5}{3}$

(c)

$x + 2 = x^2 - 4$

$x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$x = -2$ or 3

$\therefore\ (-2, 0)$ and $(3, 5)$

(d)

Susbtitution is also fine

adding

$2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$

$x = \frac{1}{2}$ or 3

$\therefore\quad x = \frac{1}{2}, y = -\frac{7}{2}$

or $\quad x = 3, y = -6$

(e)

$x = 2y$

sub.

$(2y)^2 - 4y - y^2 = 0$

$3y^2 - 4y = 0$

$y(3y - 4) = 0$

$y = 0$ or $\frac{4}{3}$

$\therefore\quad x = 0, y = 0$

or $\quad x = \frac{8}{3}, y = \frac{4}{3}$

(f)

$y = x - 5$

sub.

$x(x - 5) = 6$

$x^2 - 5x - 6 = 0$

$(x + 1)(x - 6) = 0$

$x = -1$ or 6

$\therefore\quad x = -1, y = -6$

or $\quad x = 6, y = 1$

(g)

$y = 7 - 4x$

sub.

$\frac{3}{x} - 2(7 - 4x) + 4 = 0$

$3 - 2x(7 - 4x) + 4x = 0$

$8x^2 - 10x + 3 = 0$

$(4x - 3)(2x - 1) = 0$

$x = \frac{1}{2}$ or $\frac{3}{4}$

$\therefore\quad x = \frac{1}{2}, y = 5$

or $\quad x = \frac{3}{4}, y = 4$

(h)

$4^x + 2^x = 72$

$(2)^{2x} + 2^x - 72 = 0$

$(2^x + 9)(2^x - 8) = 0$

$2^x \neq -9, 2^x = 8$

$x = 3$

$y = 8$

(i)

$3^{x-1} = (3^2)^{2y}$ $\quad\therefore\ x - 1 = 4y$

$(2^3)^{x-2} = (2^2)^{1+y}$ $\quad\therefore\ 3x - 6 = 2 + 2y$

$6x - 16 = 4y$

$\Rightarrow\quad 6x - 16 = x - 1$

$x = 3$

$\therefore\quad x = 3, y = \frac{1}{2}$

B5 Inequalities

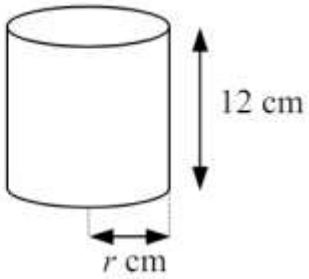
Video: https://youtu.be/wDut-In_7Wg

Solve the following inequalities:

(a)	$12 - 3x < 10$	(b)	$2(3 + x) \geq 4(6 - x)$
(c)	$x^2 - 4x + 3 < 0$	(d)	$9x - 2x^2 \leq 10$

Exam style questions

(a)



A sealed metal container for food is a cylinder of height 12 cm and base radius r cm.

Given that the surface area of the container must be at most 128π cm^2,

a show that $r^2 + 12r - 64 \leq 0$.

b Hence find the maximum value of r.

(b)

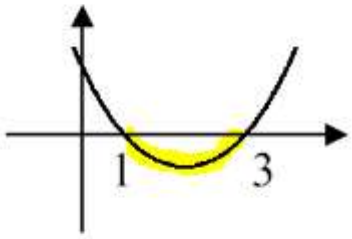
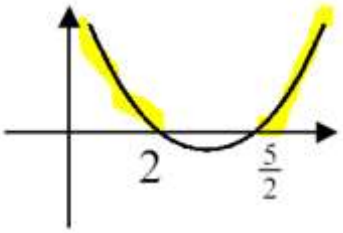
The cost for framing a picture is

- 2 pence per cm^2 of glass.
- 5 pence per cm of wooden frame.

A rectangular picture is such so that its length is 4 cm greater than its width, $x\ cm$.

If a **maximum** of £10 is available for framing, determine the range of the possible values of x.

B5 Inequalities – Answers

(a)	$2 < 3x$ $x > \frac{2}{3}$	(b)	$6 + 2x \geq 24 - 4x$ $6x \geq 18$ $x \geq 3$
(c)	$(x-1)(x-3) < 0$  $\therefore\ 1 < x < 3$	(d)	$2x^2 - 9x + 10 \geq 0$ $(2x-5)(x-2) \geq 0$  $\therefore\ x \leq 2$ or $x \geq \frac{5}{2}$

Exam style questions

(a)

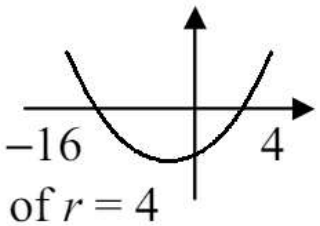
a $\text{S.A} = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 24\pi r$

$\text{S.A} \leq 128\pi \ \therefore\ 2\pi r^2 + 24\pi r \leq 128\pi$

$r^2 + 12r \leq 64$

$r^2 + 12r - 64 \leq 0$

b $(r+16)(r-4) \leq 0$

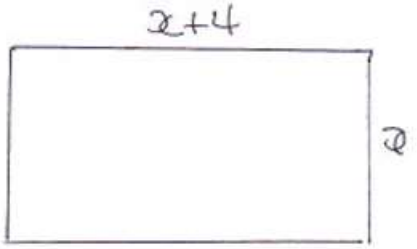


$-16 \leq r \leq 4$

$\therefore$ maximum value of $r = 4$

We will look at finding maximum values for these kinds of shapes more formally in A level Maths.

(b)



- $P = 2x + 2(x+4)$
  $P = (4x+8)$ cm
- $A = x(x+4)$
  $A = (x^2+4x)$ cm^2

COST (IN PENCE) $= 5(4x+8) + 2(x^2+4x)$

$= 20x + 40 + 2x^2 + 8x$

$= 2x^2 + 28x + 40$

≤ 10

THUS $2x^2 + 28x + 40 < 1000$

$\Rightarrow 2x^2 + 28x - 960 < 0$

$\Rightarrow x^2 + 14x - 480 < 0$

$\Rightarrow (x-16)(x+30) < 0$

6 × 80
12 × 40
24 × 20
16 × 30

C.V = 16, −30 $\Rightarrow$

$\Rightarrow \therefore 0 < x < 16$

$-30 < x < 16$

Rearranging equations

Question 1

Make a the subject $x(a - e) = d$

Question 2

Make x the subject $m(y - x) = t$

Question 3

Make x the subject of $x + a = \frac{x+b}{c}$

Question 4

Make y the subject of $y(\sqrt{3} + \sqrt{2}) = x$

and write it in the form $y = x(\sqrt{a} + \sqrt{b})$

Question 5

Make v the subject of

$$C = \frac{v^2 - ta}{x}$$

Question 6

Rearrange to make x the subject of

$$\frac{2}{x} + 5 = 6y$$

Question 7

Make y the subject of

$$\sqrt{\frac{m(y+a)}{y}} = g$$

Question 8

A cylinder has a radius of 3cm and height, h. The total surface area is $30x\ cm^2$.

Find an expression for the surface area and write h in terms of x and π.

Prove it questions

Using your rearranging skills can you prove each of the following

If $a = \frac{b}{b+c}$

Show that $\frac{a}{1-a} = \frac{b}{c}$

$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number

$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}$

Rearranging equations – Answers

Question 1

$$xa - xe = d \qquad a - e = \frac{d}{x}$$
$$xa = d + xe \quad or$$
$$a = \frac{d + xe}{x} \qquad a = \frac{d}{x} + e$$

Can you see that these are equivalent?

Question 2

$$my - mx = t$$
$$my = t + mx$$
$$mx = my - t$$
$$x = \frac{my - t}{m}$$

Question 3

$$c(x + a) = x + b$$
$$cx + ca - x = b$$
$$cx - x = b - ca$$
$$x(c - 1) = b - ca$$
$$x = \frac{b - ca}{c - 1}$$

Question 4

$$y = \frac{x}{\sqrt{3} + \sqrt{2}}$$
$$y = \frac{x}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$
$$y = \frac{x\sqrt{3} - x\sqrt{2}}{3 - 2}$$
$$y = x(\sqrt{3} - \sqrt{2})$$

Question 5

$$v^2 - ta = Cx$$
$$v^2 = Cx + ta$$
$$v = \pm\sqrt{Cx + ta}$$

Question 6

$$\frac{2}{x} = 6y - 5$$
$$x(6y - 5) = 2$$
$$x = \frac{2}{6y - 5}$$

Question 7

$$g^2 = \frac{my + ma}{y}$$
$$g^2y = my + ma$$
$$g^2y - my = ma$$
$$y(g^2 - m) = ma$$
$$y = \frac{ma}{g^2 - m}$$

Question 8

Surface area of cylinder = $2\pi r^2 + 2\pi rh$

$$30x = (2\pi \times 3^2) + (2 \times 3 \times \pi \times h)$$
$$30x = 18\pi + 6\pi h$$
$$6\pi h = 30x - 18\pi$$
$$h = \frac{30x - 18\pi}{6\pi}$$
$$h = \frac{5x - 3\pi}{\pi}$$

Prove it solutions

If $a = \frac{b}{b+c}$

Show that $\frac{a}{a-1} = \frac{b}{c}$

$\frac{a}{1} = \frac{b}{b+c}$	Make a into a fraction
$a(b+c) = b$	Using what we know about the product of the diagonals of equivalent fractions
$ab + ac = b$	Expand brackets
$ac = b - ab$	Make ac the subject
$ac = b(1-a)$	Factorise the right hand side
$b = \frac{ac}{1-a}$	Make b the subject
$\frac{b}{c} = \frac{a}{1-a}$	Divide both sides by c, expression as required

$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number

$\frac{n^2-n}{2} + \frac{n^2+n}{2}$	Expand brackets
$\frac{n^2-n+n^2+n}{2}$	Write as one fraction
$\frac{2n^2}{2}$	Simplify numerator
n^2	Cancel out factor of 2 so left with n^2 which is a square number as required

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}$$

$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6}$ — Concentrate on Left hand side

X 3, X 4, X 2

$\frac{3(2x+3)}{12} - \frac{4(3x-2)}{12} + \frac{2}{12}$ — Make a common denominator

$\frac{6x+9}{12} - \frac{12x-8}{12} + \frac{2}{12}$ — Expand brackets

$\frac{6x+9-(12x-8)+2}{12}$ — Collect terms over single denominator

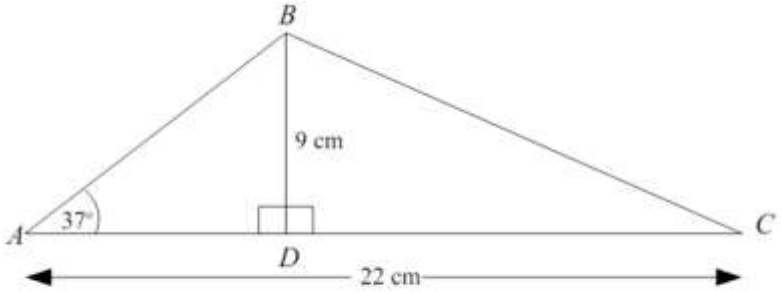
$\frac{6x+9-12x+8+2}{12}$ — Simplify

$\frac{19-6x}{12}$ — Left hand side is = to right hand side as required

E1 Triangle geometry

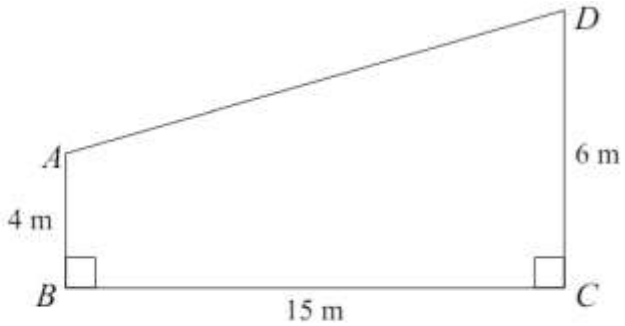
Video: https://youtu.be/uVl6TAb0vBg

Question 1



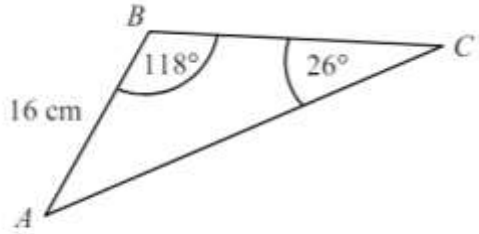
Work out the size of angle *BCD*.
Give your answer to 1 decimal place.

Question 2



Work out the size of angle *BAD*.
Give your answer to 1 decimal place.

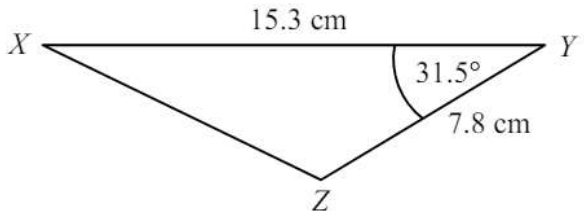
Question 3



The diagram shows triangle ABC in which $AB = 16$ cm, $\angle ABC = 118°$ and $\angle ACB = 26°$.

Find the length AC to 3 significant figures.

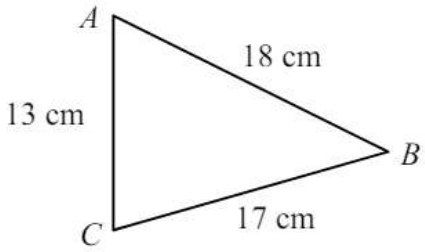
Question 4



The diagram shows triangle XYZ in which $XY = 15.3$ cm, $YZ = 7.8$ cm and $\angle XYZ = 31.5°$.

Find the length of XZ.

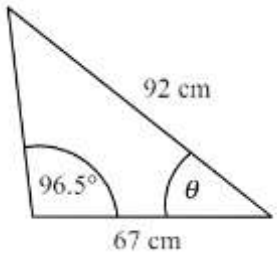
Question 5



The diagram shows triangle ABC in which $AB = 18$ cm, $AC = 13$ cm and $BC = 17$ cm.

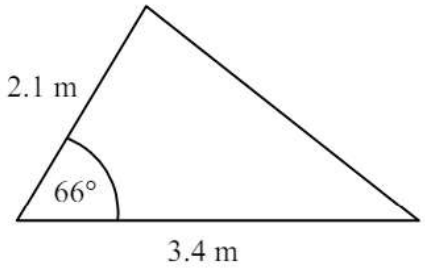
Find the size of the angle ACB

Question 6



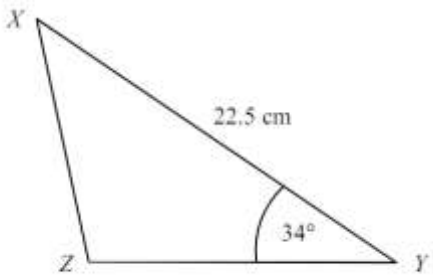
Find the angle θ

Question 7



Find the area of the triangle

Question 8



The diagram shows triangle XYZ in which $XY = 22.5$ cm and $\angle XYZ = 34°$.

Find the length of XZ

E1 Triangle geometry – Answers

Question 1

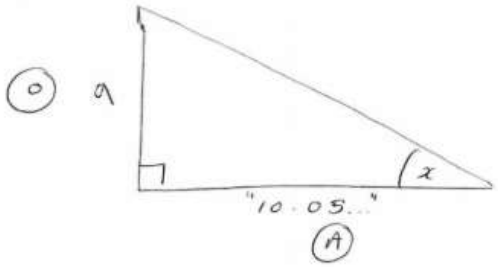
$$\tan(37) = \frac{9}{y}$$

$$y = \frac{9}{\tan(37)}$$

$$= 11.9434...$$

$$CD = 22 - 11.9434$$

$$= 10.05659...$$



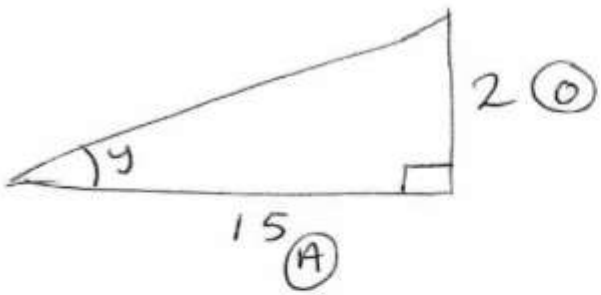
$$\tan x = \frac{9}{"10.05.."}$$

$$x = \tan^{-1}\left(\frac{9}{10.05...}\right)$$

$$= 41.8 \text{ 1dp}$$

41.8°

Question 2



$$\tan y = \frac{2}{15}$$

$$y = \tan^{-1}\left(\frac{2}{15}\right)$$

$$= 7.6^\circ \text{ (1dp)}$$

$$BAD = 90 + 7.6$$

$$= 97.6^\circ$$

Question 3

$$\frac{AC}{\sin 118} = \frac{16}{\sin 26}$$

$$AC = \frac{16 \times \sin 118}{\sin 26}$$

$$= 32.2 \text{ cm}$$

Question 4

$$XZ^2 = 7.8^2 + 15.3^2 \quad - (2 \times 7.8 \times 15.3 \times \cos 31.5°)$$

$$= 91.422$$

$$XZ = 9.56 \text{ cm (3sf)}$$

Question 5

$$18^2 = 13^2 + 17^2 - (2 \times 13 \times 17 \times \cos \angle ACB)$$

$$\cos \angle ACB = \frac{13^2 + 17^2 - 18^2}{2 \times 13 \times 17}$$

$$= 0.3032$$

$$\angle ACB = 72.4° \text{ (1dp)}$$

Question 6

$$\frac{\sin \alpha}{67} = \frac{\sin 96.5}{92}$$

$$\sin \alpha = \frac{67 \times \sin 96.5}{92}$$

$$\sin \alpha = 0.7236$$

$$\alpha = 46.351$$

$$\theta = 180 - 96.5 - \alpha$$

$$\theta = 37.1° \text{ (1dp)}$$

Question 7

area

$$= \tfrac{1}{2} \times 2.1 \times 3.4 \times \sin 66$$

$$= 3.26 \text{ m}^2 \text{ (3sf)}$$

Question 8

$$\tfrac{1}{2} \times 22.5 \times YZ \times \sin 34 = 100$$

$$YZ = \frac{200}{22.5 \times \sin 34}$$

$$= 15.896$$

$$XZ^2 = 22.5^2 + 15.896^2 - (2 \times 22.5 \times 15.896 \times \cos 34)$$

$$= 165.906$$

$$XZ = 12.9 \text{ cm (3sf)}$$

E7 Trigonometric equations

Notes and examples (if needed) – questions at the end of this section

You can of course get one solution to an equation such as sin $x = -0.5$ from your calculator. But what about others?

Example 1 Solve the equation $\sin x° = -0.5$ for $0 \leq x < 360$.

Solution The calculator gives $\sin^{-1}(0.5) = -30$.

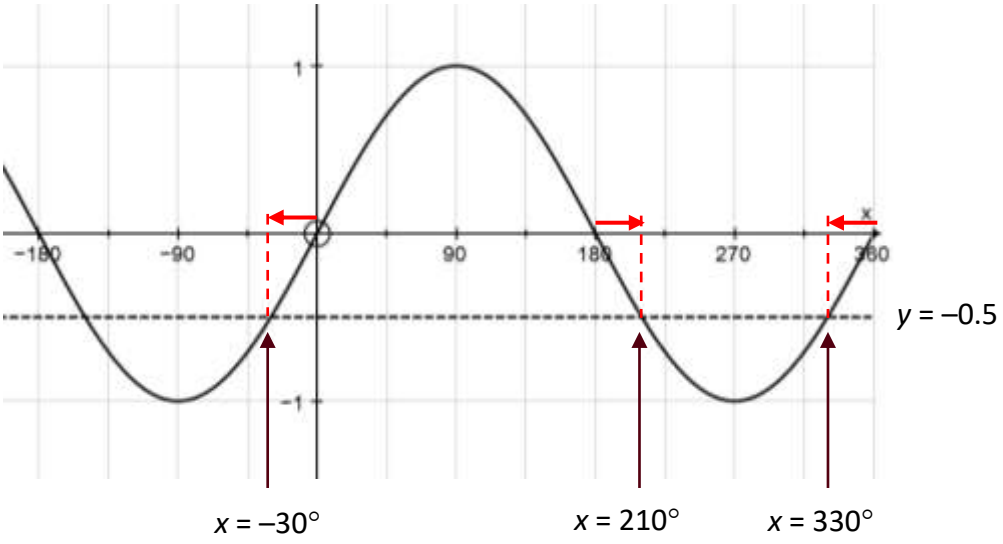
This is usually called the *principal value* of the function $\sin^{-1}$.

To get a second solution you can either use a graph or a standard rule.

Method 1: Use the graph of $y = \sin x$

By drawing the line $y = -0.5$ on the same set of axes as the graph of the sine curve, points of intersection can be identified in the range

$0 \leq x < 360$.



(The red arrows each indicate 30° to one side or the other.)

Hence the required solutions are 210° or 330°.

Method 2: Use an algebraic rule.

To find the second solution you use $\sin (180 - x)° = \sin x°$

$\tan (180 + x)° = \tan x°$

$\cos (360 - x)° = \cos x°$.

Any further solutions are obtained by adding or subtracting 360 from the principal value or the second solution.

In this example the principal solution is –30°.

Therefore, as this equation involves sine, the second solution is:

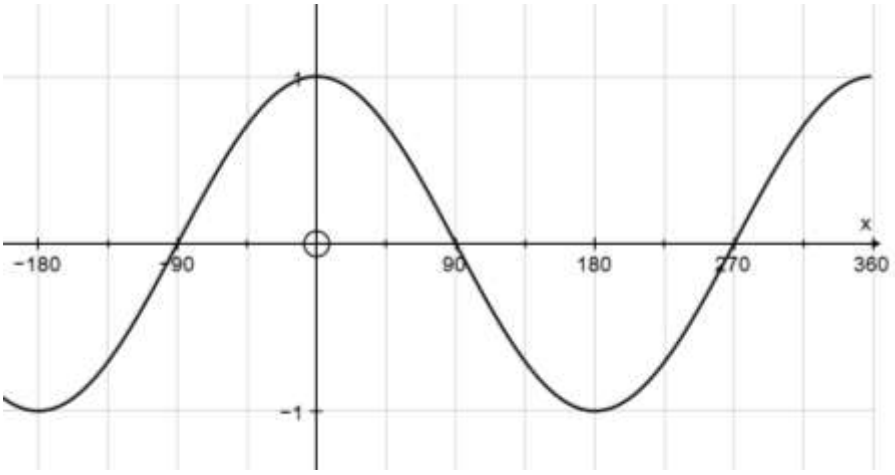
$$180 – (–30)° = 210°$$

–30° is not in the required range, so add 360 to get:

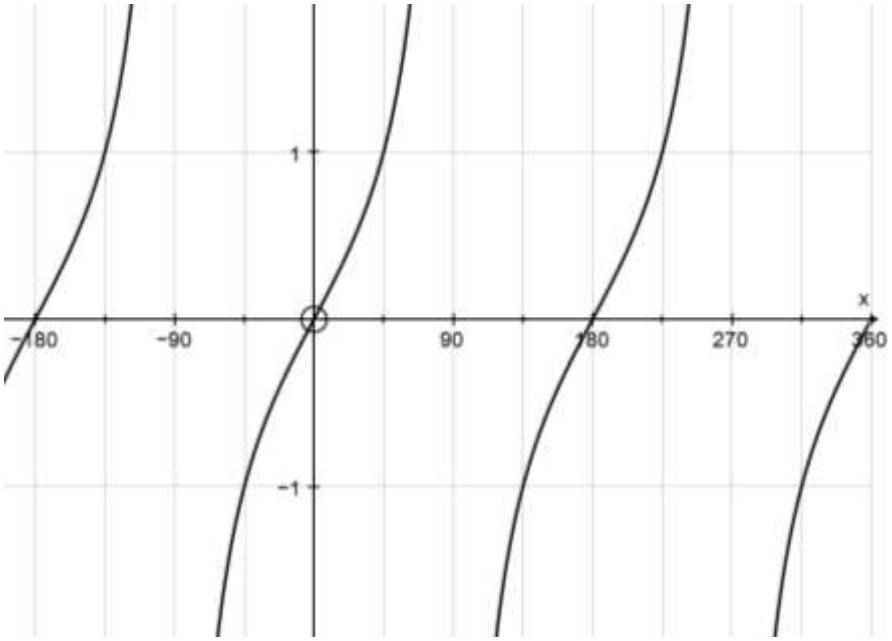
$$360 + (–30) = 330°.$$

Hence the required solutions are 210° or 330°.

You should decide which method you prefer. The corresponding graphs for cos *x* and tan *x* are shown below.



y = cos *x*



y = tan *x*

To solve equations of the form $y = \sin(kx)$, you will expect to get $2k$ solutions in any interval of 360°. You can think of compressing the graphs, or of using a wider initial range.

Example 2 Solve the equation $\sin 3x° = 0.5$ for $0 \leq x < 360$.

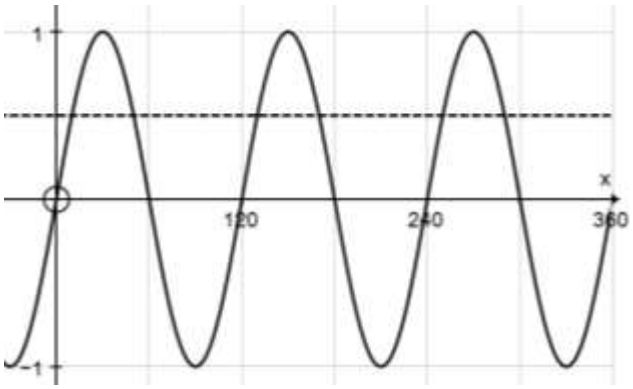
Solution ***Method 1:*** Use the graph.

The graph of $y = \sin 3x°$ is the same as the graph of $y = \sin x°$ but compressed by a factor of 3 (the *period* is 120°).

The calculator gives $\sin^{-1}(0.5) = 30$, so the principal solution is given by

$$3x = 30 \Rightarrow x = 10.$$

The vertical lines on the graph below are at multiples of 60°. So you can see from the graph that the other solutions are 50°, 130°, 170°, 250° and 290°.



$y = 0.5$

$y = \sin 3x$

Method 2: The principal value of $3x$ is $\sin^{-1}(0.5) = 30°$.

Therefore $3x = 30$ or $180 - 30 = 150$,

or $360 + 30$ or $360 + 150$

or $720 + 30$ or $720 + 150$

$\Rightarrow 3x = 30, 150, 390, 510, 750, 870$

$\Rightarrow x = 10, 50, 130, 170, 250, 290.$

Notice that with Method 2 you have to look at values of $3x$ in the range 0 to 1080 ($= 3 \times 360$), which is somewhat non-intuitive.

Questions

Question 1

Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1°.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

Question 2

Solve the following equations for $-180 \leq x < 180$. Give your answers to the nearest 0.1°.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

Question 3

Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1°.

(a) $\sin 2x° = 0.829$ (b) $\cos 3x° = 0.454$ (c) $\tan 4x = 2.05$

(d) $\sin \frac{1}{2}x° = 0.8$ (e) $\cos \frac{1}{2}x° = 0.3$ (f) $\tan \frac{1}{3}x° = 0.7$

E7 Trigonometric equations – Answers

Question 1

(a) 64.2, 115.8 (b) 53.1, 306.9 (c) 63.4, 243.4

(d) 203.6, 336.4 (e) 120, 240 (f) 108.4, 288.4

Question 2

(a) 64.2, 115.8 (b) 53.1, –53.1 (c) 63.4, –116.6

(d) –23.6, –156.4 (e) 120, –120 (f) –71.5, 108.4

Question 3

(a) 28, 62, 208, 242

(b) 21, 99, 141, 219, 261, 339

(c) 16, 61, 106, 151, 196, 241, 286, 331

(d) 106.2, 253.7

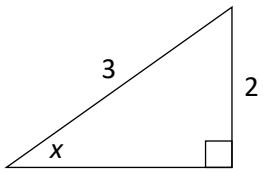
(e) 145.1

(f) 105

E3 Exact trigonometric values

Notes and examples (if needed) – questions at the end of this section

Suppose that you are told that sin $x°$ is exactly $\frac{2}{3}$. Assuming that x is between 0° and 90°, you can find the exact values of cos $x°$ and tan $x°$ by drawing a right-angled triangle in which the opposite side and the hypotenuse are 2 and 3 respectively:



Now Pythagoras's Theorem tells you that the third, adjacent, side is $\sqrt{3^2-2^2}=\sqrt{5}$.

Hence using SOH, CAH, TOAH, cos $x°$ = $\frac{\sqrt{5}}{3}$ and tan $x°$ = $\frac{2}{\sqrt{5}}$.

This is preferable to using a calculator as the calculator does not always give exact values for this type of calculation. (Calculators can *in general* not handle irrational numbers exactly, although many are programmed to do so in simple cases.)

Questions

Do not use a calculator in this exercise.

In this question θ is in the range $0 \le \theta < 90$.

(a) Given that $\sin\theta = \frac{12}{13}$, find the exact values of cos θ and tan θ.

(b) Given that $\tan\theta = \frac{6}{7}$, find the exact values of sin θ and cos θ.

(c) Given that $\cos\theta = \frac{5}{8}$, find the exact values of sin θ and tan θ.

E3 Exact trigonometric values – Answers

(a) $\frac{5}{13}$, $\frac{12}{5}$ (b) $\frac{6}{\sqrt{85}}$, $\frac{7}{\sqrt{85}}$ (c) $\frac{\sqrt{39}}{8}$, $\frac{\sqrt{39}}{5}$

Practice Test 1

Write out the solutions to each of the following questions. Show full working, without the use of a calculator.

B1 Indices

1.	Evaluate $\left(\frac{8}{125}\right)^{-2/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$	**3.**	Solve $9^{x-2} = 27$	**4.**	Solve $16^x = 4^{1-x}$

B2 Surds

1.	Simplify $\sqrt{72}$	**2.**	Expand and simplify $(2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$	**3.**	Rationalise the denominator $\frac{11}{2\sqrt{5}}$	**4.**	Rationalise the denominator $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}}$

B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.

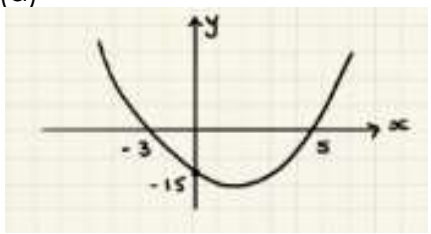
(a) (i) $x^2 + 3x - 28 = 0$	(b) (i) $x^2 - 6x + 9 = 0$	(c) (i) $2x^2 - 21x + 27 = 0$
(a) (ii) Sketch $y = x^2 + 3x - 28$	(b) (ii) Sketch $y = x^2 - 6x + 9$	(c) (ii) Sketch $y = 2x^2 - 21x + 27$

2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.

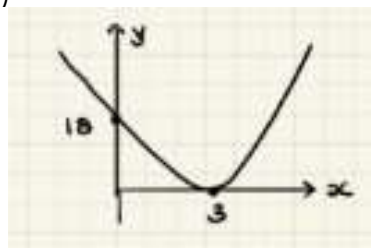
(a) (i) $x^2 + 4x - 7 = 0$	(b) (i) $11 + 8x - x^2 = 0$	(c) (i) $3x^2 - 12x + 2 = 0$
(ii) Write $y = x^2 + 4x - 7$ in the form $y = a(x + b)^2 + c$	(ii) Write $y = 11 + 8x - x^2$in the form $y = a(x + b)^2 + c$	(ii) Write $y = 3x^2 - 12x + 2$ in the form $y = a(x + b)^2 + c$
(iii) Sketch $y = x^2 + 4x - 7$	(iii) Sketch $y = 11 + 8x - x^2$	(iii) Sketch $y = 3x^2 - 12x + 2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$

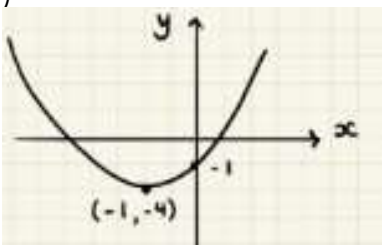
(a)



(b)



(c)



B4 Simultaneous Equations

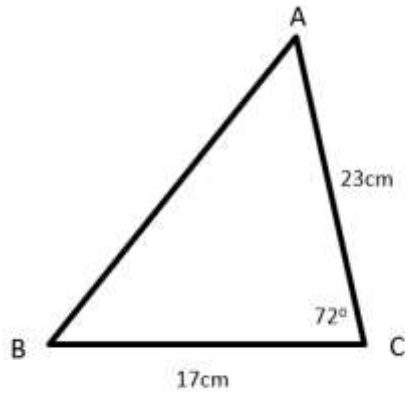
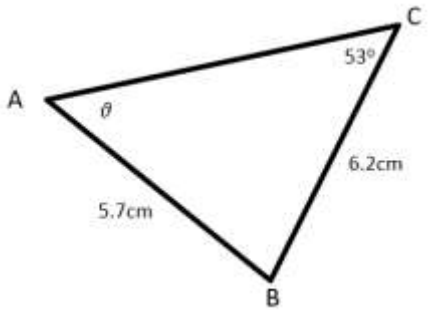
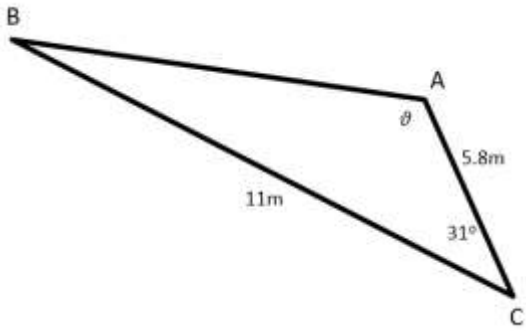
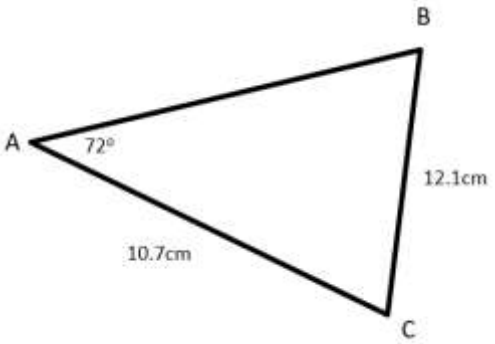
1.	Solve $3x + 3y = -4$ $5x - 2y = 5$	**2.**	Solve $y = x - 6$ $\frac{1}{2}x - y = 4$	**3.**	Solve $3x^2 - x - y^2 = 0$ $x + y = 1$

B5 Inequalities

Find the set of values for which…

1.	$3(1-2t) \leq t-4$	**2.**	$2x^2 - 9x + 4 \leq 0$	**3.**	$2y + 3 < 3y(y-2)$

E1 Triangle Geometry (Calculator)

1.	Calculate the length AB 	**2.**	Calculate the angle ϑ 
3.	Calculate the length AB and the obtuse angle ϑ 	**4.**	Calculate the area of the triangle ABC 

Rearranging equations

1.	To find velocity, v, we use the formula $v^2 = u^2 + 2as$ Rearrange to find s	**2.**	Make x the subject of $4F = F + \frac{a}{y+x}$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \leq \theta \leq 360°$ giving your answers to 1 decimal place.			
1.	$\cos\theta = 0.4$	**2.**	$\sin 2x° = 0.5$

E3 Exact trigonometric values

Find the exact values of $\cos x$ and $\tan x$ given that:

$\sin x = \frac{4}{5}$ and $0° < x < 90°$

Practice Test 1 – Answers

Practice 1

B1 Indices

1. $\left(\frac{8}{125}\right)^{-2/3}$

$= \left(\frac{125}{8}\right)^{2/3}$

$= \left(\frac{5}{2}\right)^{2}$ M1

$= \frac{25}{4}$ A1

2. $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/3}}{x^2}$ M1

$= \frac{x^{5/6}}{x^2}$ A1

$= x^{-7/6}$ A1

3. $9^{x-2} = 27$

$(3^2)^{x-2} = 3^3$ M1

$3^{2x-4} = 3^3$

$2x - 4 = 3$ M1

$2x = 7$

$x = 7/2$ A1

4. $16^{x} = 4^{1-x}$

$(4^2)^{x} = 4^{1-x}$ M1

$4^{2x} = 4^{1-x}$ [1]

$2x = 1 - x$ M1

$3x = 1 \Rightarrow x = 1/3$ A1

B2 Surds

1. $\sqrt{72}$

$= \sqrt{36 \times 2}$

$= 6\sqrt{2}$ A1

2. $(2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$

$42 + 8\sqrt{21} - 15\sqrt{21} - 60$ m1 A1

$-7\sqrt{21} - 18$ A1

3. $\frac{11}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ M1

$= \frac{11\sqrt{5}}{10}$ A1

4. $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}} \quad \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$ M1

$= \frac{16 - 8\sqrt{5} - 6\sqrt{5} + 15}{4 - 5}$ A1 A1

$= \frac{31 - 14\sqrt{5}}{-1} = 14\sqrt{5} - 31$ A1

[10]

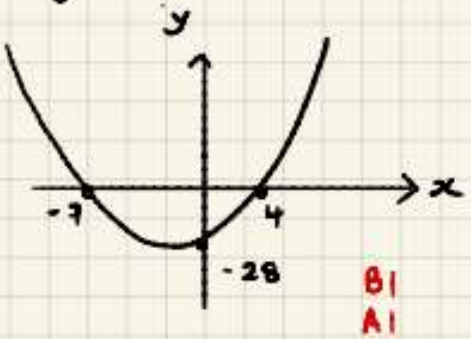
B3 Quadratics

1. (a) (i) $x^2 + 3x - 28 = 0$

$(x + 7)(x - 4) = 0$ M1

$x = -7$ or $x = 4$ A1

(ii) $y = x^2 + 3x - 28$



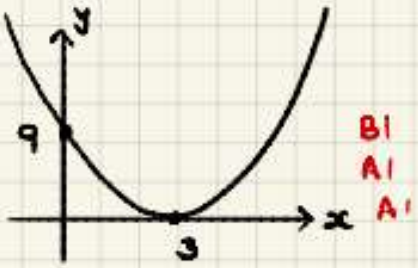
B1
A1
A1

(b) (i) $x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$ M1

A1 $x = 3$ (repeated)

(ii) $y = x^2 - 6x + 9$



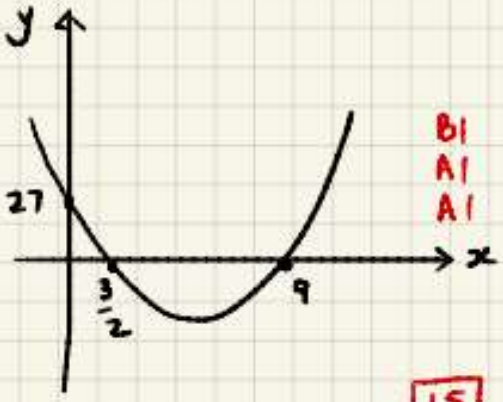
B1
A1
A1

(c) (i) $2x^2 - 21x + 27 = 0$

$(2x - 3)(x - 9) = 0$ M1

$x = 3/2$ $x = 9$ A1

(ii) $y = 2x^2 - 21x + 27$



B1
A1
A1

15

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

2. (a) (i) $x^2 + 4x - 7 = 0$

$(x+2)^2 - 4 - 7 = 0$ M1

$(x+2)^2 = 11$

$x + 2 = \pm\sqrt{11}$

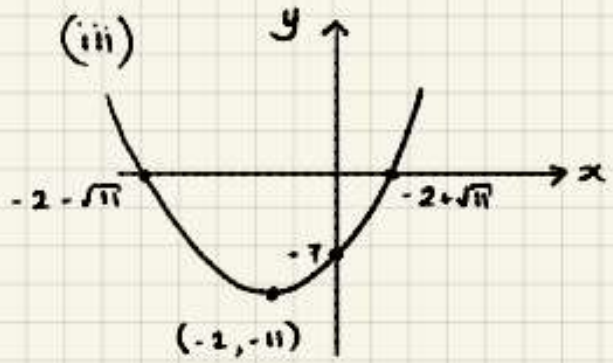
$x = -2 \pm \sqrt{11}$ A1

Graphs
- B1 Shape
- A1 Vertex
- A1 intersections x-axis
- A1 intersections y-axis

(ii) $y = x^2 + 4x - 7$

$y = (x+2)^2 - 11$ B1

(iii)



(b) (i) $11 + 8x - x^2 = 0$

$-(x^2 - 8x - 11) = 0$ M1

$-[(x-4)^2 - 16 - 11] = 0$ M1

$-(x-4)^2 + 27 = 0$

$(x-4)^2 = 27$

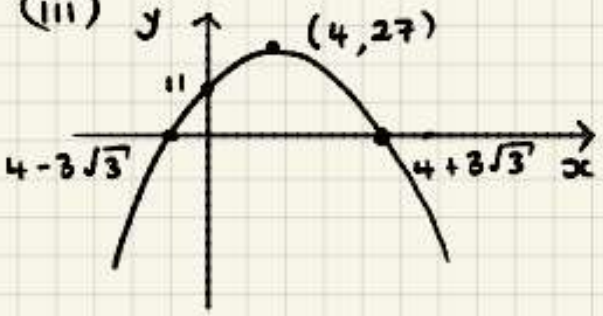
$x - 4 = \pm 3\sqrt{3}$

$x = 4 \pm 3\sqrt{3}$ A1

(ii) $y = 11 + 8x - x^2$

$y = 27 - (x-4)^2$ B1

(iii)



(c) (i) $3x^2 - 12x + 2 = 0$

$3\left[x^2 - 4x + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - 4 + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - \frac{10}{3}\right] = 0$

$3(x-2)^2 - 10 = 0$

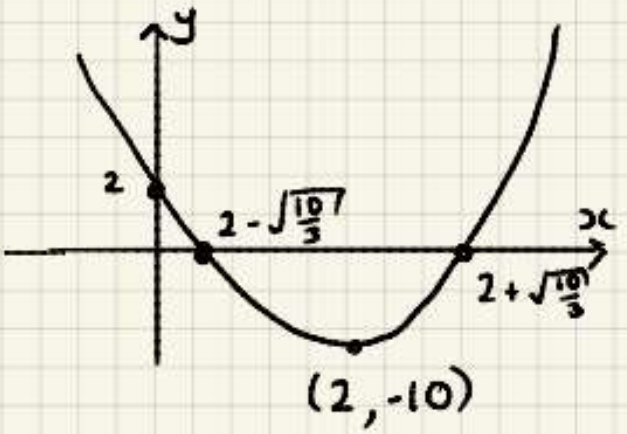
$(x-2)^2 = \frac{10}{3}$

$x - 2 = \pm\sqrt{\frac{10}{3}}$

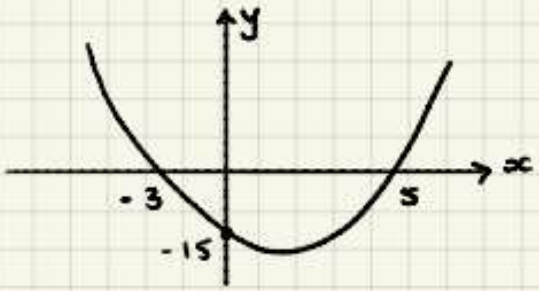
$x = 2 \pm \sqrt{\frac{10}{3}}$ A1

(ii) $y = 3x^2 - 12x + 2$

$y = 3(x-2)^2 - 10$ B1



3. (a)



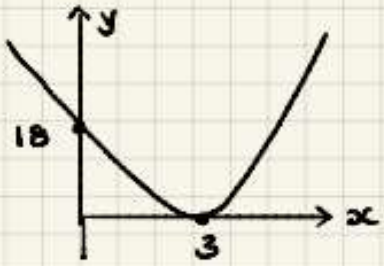
$y = k(x+3)(x-5)$ M1

$-15 = k(3)(-5) \Rightarrow k = 1$ A1

$y = (x+3)(x-5)$

$y = x^2 - 2x - 15$ A1

(b)



$y = k(x-3)^2$ M1

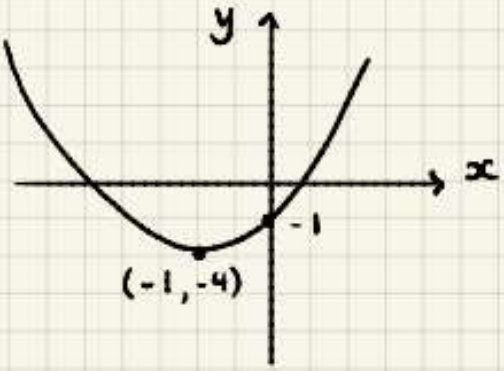
$18 = k(-3)^2 \Rightarrow k = 2$ A1

$y = 2(x-3)^2$

$y = 2(x^2 - 6x + 9)$

$y = 2x^2 - 12x + 18$ A1

(c)



$y = k(x+1)^2 - 4$ M1

(0, -1)

$-1 = k(1)^2 - 4$

$\Rightarrow k = 3$ A1

$y = 3(x+1)^2 - 4$

$y = 3(x^2 + 2x + 1) - 4$

$y = 3x^2 + 6x - 1$ A1

[9]

04 Simultaneous Equations

1. $3x + 3y = -4$ $\quad 6x + 6y = -8$

$5x - 2y = 5$ $\quad 15x - 6y = 15$ added M1

$21x = 7$

$x = 1/3$ A1 $\quad 3(1/3) + 3y = -4$

$3y = -5$

$x = 1/3, \ y = -5/3$ A1

2. $y = x - 6$

$\frac{1}{2}x - y = 4$

$\frac{1}{2}x - (x - 6) = 4$ M1

$\frac{1}{2}x - x + 6 = 4$

$-\frac{1}{2}x = -2$

$x = 4$ A1 $\quad y = 4 - 6$

$y = -2$

$x = 4, \ y = -2$ A1

3. $3x^2 - x - y^2 = 0$ $\quad x + y = 1$

$3x^2 - x - (1-x)^2 = 0$ M1 $\quad y = 1 - x$

$3x^2 - x - (1 - 2x + x^2) = 0$

$3x^2 - x - 1 + 2x - x^2 = 0$

$2x^2 + x - 1 = 0$ A1

$(2x - 1)(x + 1) = 0$

$x = 1/2$ $\quad x = -1$ A1

$y = 1 - 1/2$ $\quad y = 1 - -1$

$x = 1/2$ $y = 1/2$ A1 $\quad x = -1$, $y = 2$ A1

11

B5 Inequalities

1. $3(1 - 2t) \leq t - 4$

$3 - 6t \leq t - 4$

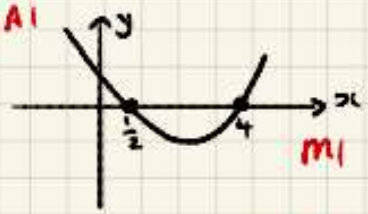
$7 \leq 7t$ M1

$t \geq 1$ A1

2. $2x^2 - 9x + 4 \leq 0$

$(2x - 1)(x - 4) \leq 0$ M1

CVs $x = 1/2$ $\quad x = 4$ A1



M1

$\frac{1}{2} \leq x \leq 4$ A1

3. $2y + 3 < 3y(y - 2)$

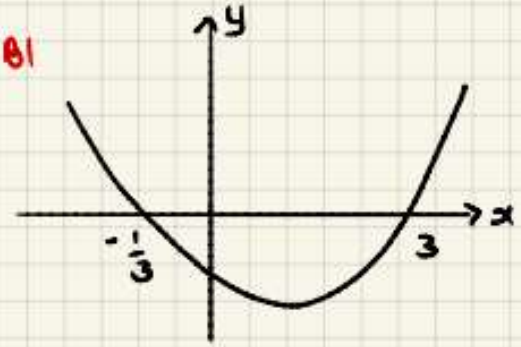
$2y + 3 < 3y^2 - 6y$

$0 < 3y^2 - 8y - 3$ B1

$3y^2 - 8y - 3 > 0$

$(3y + 1)(y - 3) > 0$ M1

CVs $y = -1/3$ $\quad y = 3$ A1



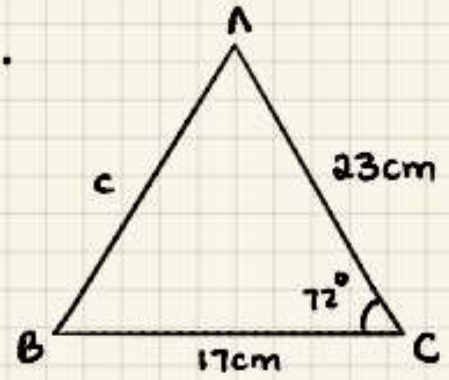
M1

11

$y < -1/3$ or $y > 3$ A1

E1 Triangle Geometry

1.



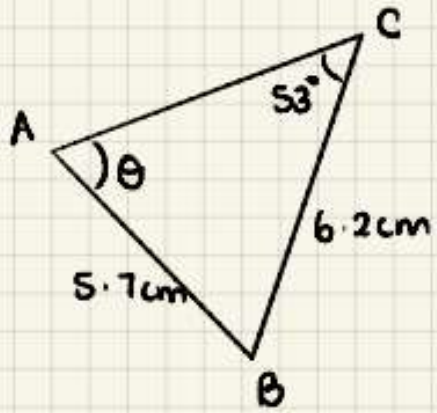
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 17^2 + 23^2 - 2(17)(23)\cos 72^\circ$

$c^2 = 576.35$ M1

$AB = 24.0\text{cm}$ A1

2.

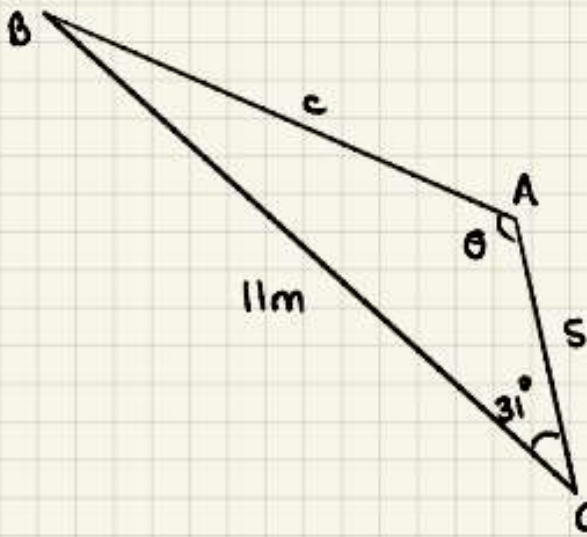


$\frac{\sin\theta}{6.2} = \frac{\sin 53}{5.7}$ M1

$\theta = \sin^{-1}\left(\frac{6.2\sin 53}{5.7}\right)$

$\theta = 60.3^\circ$ A1

3.



AB

$c^2 = 5.8^2 + 11^2 - 2(5.8)(11)\cos 31$

$c^2 = 45.27$ M1

$AB = 6.7\text{m}$ A1

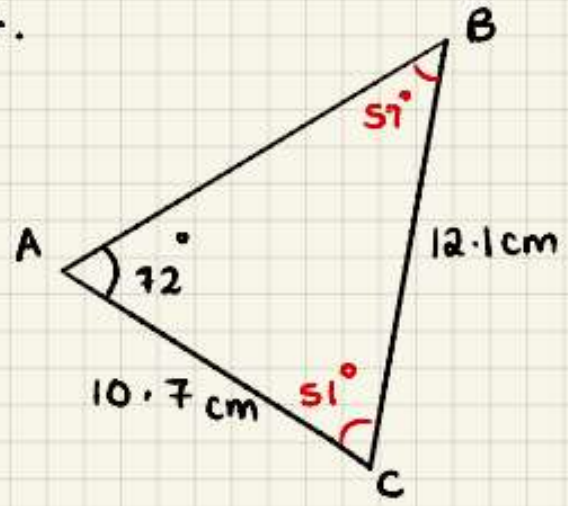
θ

$\cos\theta = \frac{5.8^2 + 6.7^2 - 11^2}{2(5.8)(6.7)}$ M1

$\theta = \cos^{-1}(-0.546)$ A1

$\theta = 123^\circ$

4.



$$\frac{\sin B}{10.7} = \frac{\sin 72^\circ}{12.1}$$ M1

$$B = \sin^{-1}\left(\frac{10.7 \sin 72^\circ}{12.1}\right)$$

$$\theta = 57^\circ$$ A1

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(10.7)(12.1) \sin 51^\circ$$ M1

$$= 50.3 \text{cm}^2$$ A1

12

Rearranging equations

1.

To find velocity, v, we use the formula
$v^2 = u^2 + 2as$
Rearrange to find s

$$v^2 - u^2 = 2as$$
$$s = \frac{v^2 - u^2}{2a}$$

2.

Make x the subject of

$$4F = F + \frac{a}{y + x}$$

$$3F = \frac{a}{y + x}$$
$$3Fy + 3Fx = a$$
$$3Fx = a - 3Fy$$
$$x = \frac{a - 3FY}{3F}$$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \leq \theta \leq 360°$ giving your answers to 1 decimal place.

1.

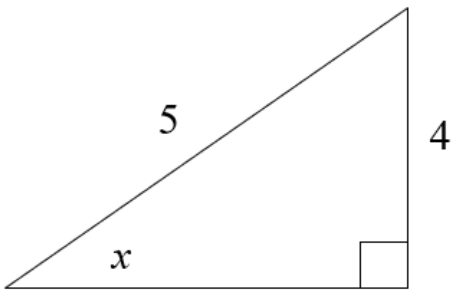
$$\theta = 66.4, 360 - 66.4$$
$$\theta = 66.4°, 293.6°$$

2.

$$2x = 30, 180 - 30,$$
$$360 + 30, 540 - 30$$
$$= 30, 150, 390, 510$$
$$x = 15, 75, 195, 255$$

E3 Exact trigonometric values

1.



Now Pythagoras's Theorem tells you that the third, adjacent, side is 3 $\sqrt{5^2 - 4^2} = 3$

Hence using SOH, CAH, TOA, $\cos x° = \frac{3}{5}$

$$\tan x° = \frac{4}{3}$$

Practice Test 2

Write out the solutions to each of the following questions. Show full working, without the use of a calculator.

B1 Indices

1.	Evaluate $\left(3\frac{3}{8}\right)^{-1/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x} \times \sqrt[5]{x}}{x^2}$	**3.**	Solve $3^{3x-2} = \sqrt[3]{9}$	**4.**	Solve $\left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{8}\right)^{2x}$

B2 Surds

1.	Simplify $\sqrt{80}$	**2.**	Expand and simplify $(7-3\sqrt{5})(3\sqrt{5}-2)$	**3.**	Rationalise the denominator $\frac{7}{5\sqrt{3}}$	**4.**	Rationalise the denominator $\frac{3+5\sqrt{11}}{7-\sqrt{11}}$

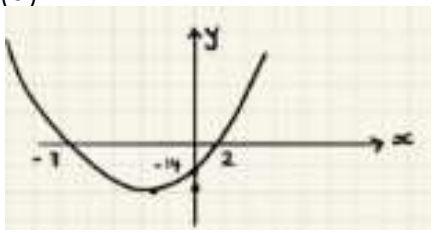
B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.		
(a) (i) $x^2 - 13x + 40 = 0$	(b) (i) $x^2 + 5x = 0$	(c) (i) $6x^2 + 5x - 4 = 0$
(a) (ii) Sketch $y = x^2 - 13x + 40$	(b) (ii) Sketch $y = x^2 + 5x$	(c) (ii) Sketch $y = 6x^2 + 5x - 4$

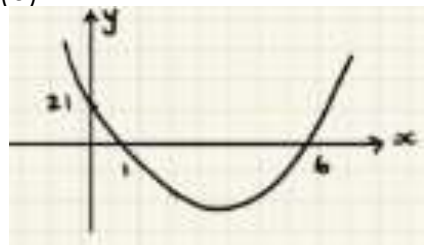
2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.		
(a) (i) $x^2 + 2x - 20 = 0$	(b) (i) $-11 + 8x - x^2 = 0$	(c) (i) $3x^2 - 18x + 2 = 0$
(ii) Write $y = x^2 + 2x - 20$ in the form $y = a(x + b)^2 + c$	(ii) Write $y = -11 + 8x - x^2$in the form $y = a(x + b)^2 + c$	(ii) Write $y = 3x^2 - 18x + 2$ in the form $y = a(x + b)^2 + c$
(iii) Sketch $y = x^2 + 2x - 20$	(iii) Sketch $y = -11 + 8x - x^2$	(iii) Sketch $y = 3x^2 - 18x + 2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$

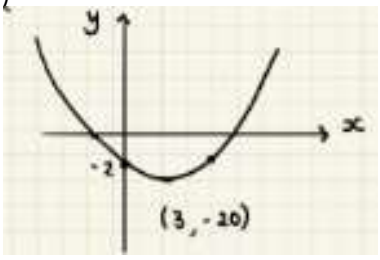
(a)



(b)



(c)



B4 Simultaneous Equations

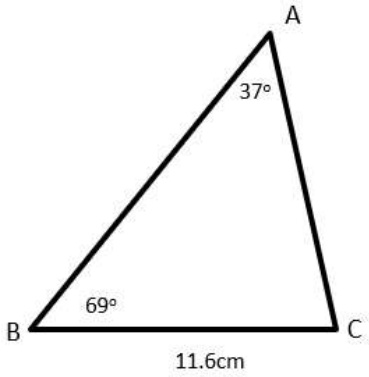
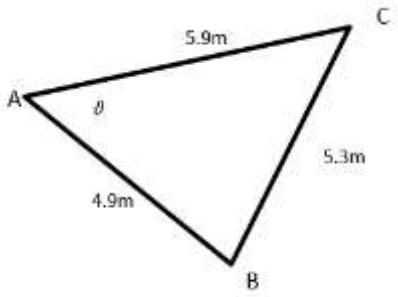
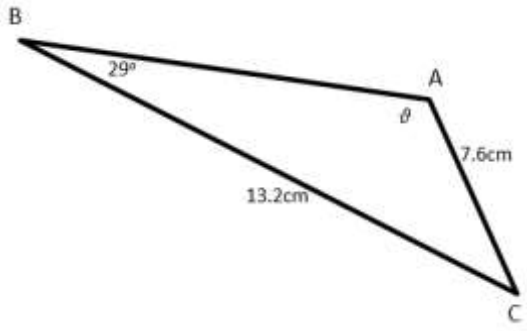
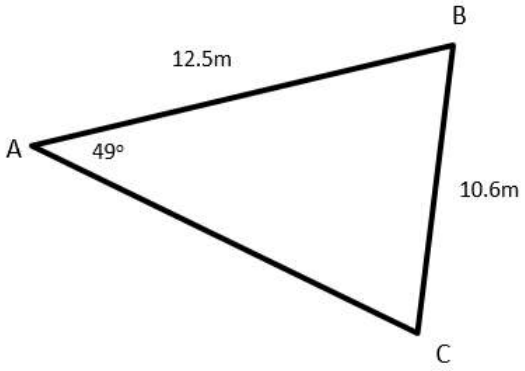
1.	Solve $3x - 4y = 16$ $2x + 12y = 7$	**2.**	Solve $3y = 2x - 8$ $4x + y = -5$	**3.**	Solve $3x^2 - xy + y^2 = 36$ $x - 2y = 10$

B5 Inequalities

Find the set of values for which…

1.	$4(5-2y) \geq 3(7-2y)$	**2.**	$2x^2 - 5x - 3 > 0$	**3.**	$x(2x+1) \leq x^2 + 6$

E1 Triangle Geometry (Calculator)

1.	Calculate the length AB 	**2.**	Calculate the angle ϑ 
3.	Calculate the length AB and the obtuse angle ϑ 	**4.**	Calculate the area of the triangle ABC 

Rearranging equations

1.	Make x the subject of $x + a = \frac{x+b}{c}$	**2.**	Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \leq \theta \leq 360°$ giving your answers to 1 decimal place.			
1.	$\tan\theta = 1.6$	**2.**	$\cos 2x° = 0.64$

E3 Exact trigonometric values

Find the exact values of $\cos x$ and $\tan x$ given that:

$$\sin x = \frac{1}{3} \text{ and } 0° < x < 90°$$

Practice Test 2 – Answers

Practice Test 2

B1 Indices

1. $\left(3\frac{3}{8}\right)^{-1/3} = \left(\frac{27}{8}\right)^{-\frac{1}{3}}$ M1

$= \left(\frac{8}{27}\right)^{1/3}$

$= \frac{2}{3}$ A1

2. $\frac{\sqrt{x} \times \sqrt[5]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/5}}{x^2}$ M1

$= \frac{x^{7/10}}{x^2}$ A1 $= x^{-13/10}$ A1

3. $3^{3x-2} = \sqrt[3]{9}$

$3^{3x-2} = (3^2)^{1/3}$ M1

$3^{3x-2} = 3^{2/3}$

$3x - 2 = \frac{2}{3}$ M1

$3x = \frac{8}{3} \Rightarrow x = \frac{8}{9}$ A1

4. $\left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{8}\right)^{2x}$

$(2^{-1})^{1-x} = (2^{-3})^{2x}$ M1

$2^{-1+x} = 2^{-6x}$

$-1 + x = -6x$ M1

$7x = 1$

$x = 1/7$ A1

11

B2 Surds

1. $\sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$ A1

2. $(7 - 3\sqrt{5})(3\sqrt{5} - 2)$

$= 21\sqrt{5} - 14 - 45 + 6\sqrt{5}$ M1 A1

$= 27\sqrt{5} - 59$ A1

3. $\frac{7}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ M1

$= \frac{7\sqrt{3}}{15}$ A1

4. $\frac{3 + 5\sqrt{11}}{7 - \sqrt{11}} \quad \frac{(7 + \sqrt{11})}{(7 + \sqrt{11})}$ M1

$= \frac{21 + 8\sqrt{11} + 55}{49 - 11}$ A1 A1

$= \frac{76 + 38\sqrt{11}}{38}$

$= 2 + \sqrt{11}$ A1

10

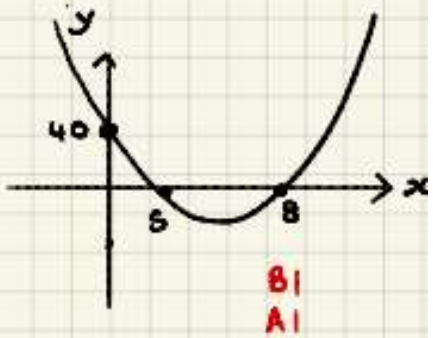
B3 Quadratics

1. (a) (i) $x^2 - 13x + 40 = 0$

$(x-8)(x-5) = 0$ M1

$x = 8 \quad x = 5$ A1

(ii) $y = x^2 - 13x + 40$



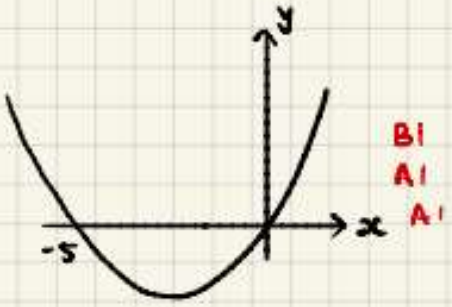
B1
A1
A1

(b) (i) $x^2 + 5x = 0$

$x(x+5) = 0$ M1

$x = 0 \quad x = -5$ A1

(ii) $y = x^2 + 5x$



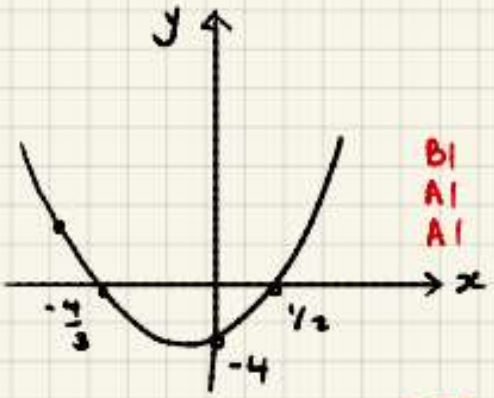
B1
A1
A1

(c) (i) $6x^2 + 5x - 4 = 0$

$(3x+4)(2x-1) = 0$ M1

$x = -4/3 \quad x = 1/2$ A1

(ii) $y =$



B1
A1
A1

15

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

2. (a) (i) $x^2 + 2x - 20 = 0$

$(x+1)^2 - 1 - 20 = 0$ M1

$(x+1)^2 = 21$

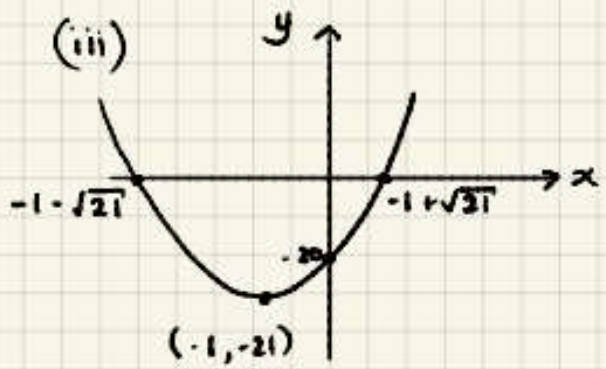
$x+1 = \pm\sqrt{21}$

$x = -1 \pm \sqrt{21}$ A1

(ii) $y = x^2 + 2x - 20$

$y = (x+1)^2 - 21$ B1

(iii)



Graphs
- B1 Shape
- A1 Vertex
- A1 intersections x-axis
- A1 intersections y-axis

(b) (i) $-11 + 8x - x^2 = 0$

$-(x^2 - 8x + 11) = 0$ M1

$-[(x-4)^2 - 16 + 11] = 0$ M1

$5 - (x-4)^2 = 0$

$(x-4)^2 = 5$

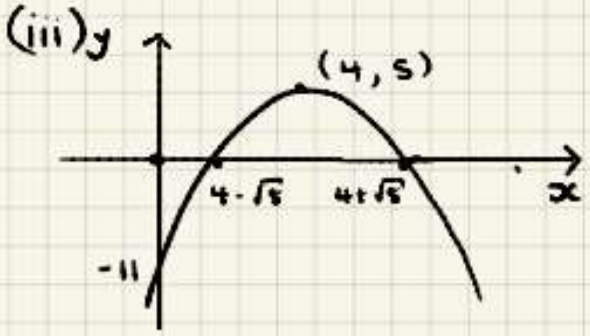
$x - 4 = \pm\sqrt{5}$

$x = 4 \pm \sqrt{5}$ A1

(ii) $y = -11 + 8x - x^2$

$y = 5 - (x-4)^2$ B1

(iii)



(c) (i) $3x^2 - 18x + 2 = 0$

$3\left[x^2 - 6x + \frac{2}{3}\right] = 0$ M1

$3\left[(x-3)^2 - 9 + \frac{2}{3}\right] = 0$ M1

$3\left[(x-3)^2 - \frac{25}{3}\right] = 0$

$3(x-3)^2 - 25 = 0$

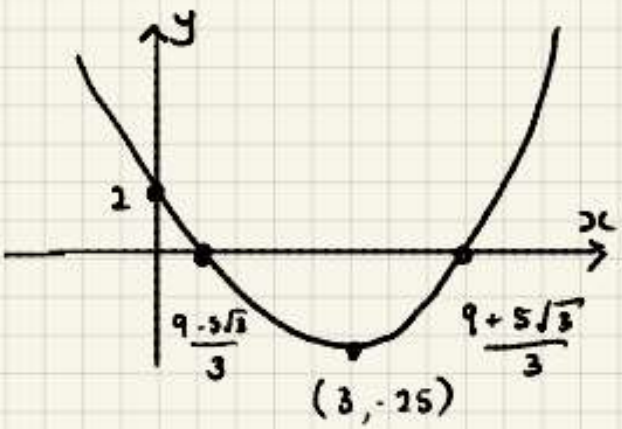
$3(x-3)^2 = 25$

$x - 3 = \pm\frac{5}{\sqrt{3}}$

$x = \frac{9 \pm 5\sqrt{3}}{3}$ A1

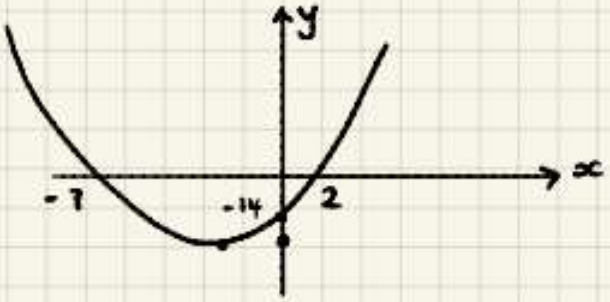
(ii) $y = 3x^2 - 18x + 2$

$y = 3(x-3)^2 - 25$ B1



23

3. (a)



$y = k(x+7)(x-2)$ M1

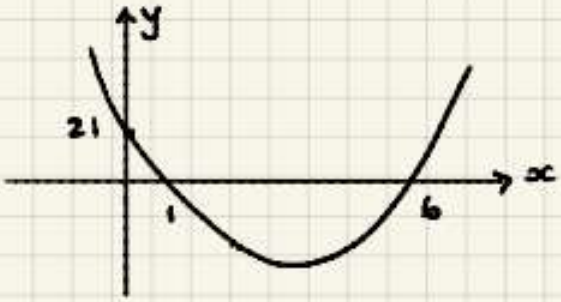
$-14 = k(7)(-2)$

$k = 1$ A1

$y = (x+7)(x-2)$

$y = x^2 + 5x - 14$ A1

(b)



$y = k(x-1)(x-6)$ M1

$21 = k(-1)(-6)$

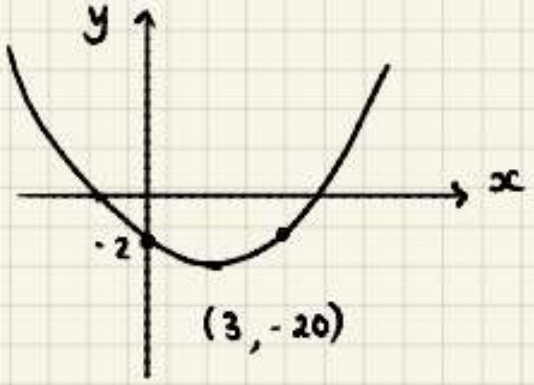
$\Rightarrow k = \frac{21}{6} = \frac{7}{2}$ A1

$y = \frac{7}{2}(x-1)(x-6)$

$y = \frac{7}{2}(x^2 - 7x + 6)$

$y = \frac{7x^2}{2} - \frac{49x}{2} + 21$ A1

(c)



$y = k(x-3)^2 - 20$ M1

$-2 = k(-3)^2 - 20$

$18 = k(9)$

$k = 2$ A1

$y = 2(x-3)^2 - 20$

$y = 2(x^2 - 6x + 9) - 20$

$y = 2x^2 - 12x - 2$ A1

[9]

B4. Simultaneous Equations

1. $3x - 4y = 16$ $\quad 9x - 12y = 48$

$2x + 12y = 7$ $\quad 2x + 12y = 7$ M1

$11x = 55$

$x = 5$ A1

$3x - 4y = 16$
$15 - 4y = 16$
$-1 = 4y$
$y = -1/4$

$x = 5, y = -1/4$ A1

2. $3y = 2x - 8 \Rightarrow 2x = 3y + 8$
$4x = 6y + 16$

$4x + y = -5$ M1

$6y + 16 + y = -5$

$7y = -21$

$y = -3$

$2x = 3y + 8$
$2x = 3(-3) + 8$
$x = -1/2$ A1 $\quad x = -1/2$ $y = -3$ A1

3. $3x^2 - xy + y^2 = 36$

$x - 2y = 10 \Rightarrow x = 2y + 10$

$3(2y+10)^2 - (2y+10)y + y^2 = 36$ M1

$3(4y^2 + 40y + 100) - y(2y+10) + y^2 = 36$

$12y^2 + 120y + 300 - 2y^2 - 10y + y^2 = 36$

$11y^2 + 110y + 264 = 0$

$y^2 + 10y + 24 = 0$ A1

$(y+6)(y+4) = 0$

M1

$y = -6$ $\quad y = -4$

$x = 2(-6) + 10$ $\quad x = 2(-4) + 10$

$x = -2$ $\quad x = 2$

$x = -2, y = -6$ A1 $\quad x = 2, y = -4$ A1

11

BS Inequalities

1. $4(5-2y) \geqslant 3(7-2y)$

$20 - 8y \geqslant 21 - 6y$ M1

$-1 \geqslant 2y$

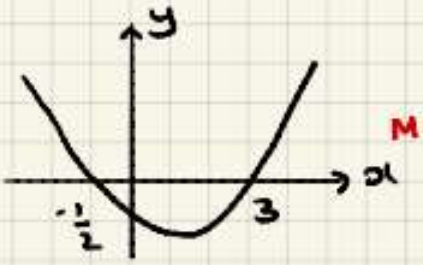
$-\frac{1}{2} \geqslant y$

$y \leqslant -\frac{1}{2}$ A1

2. $2x^2 - 5x - 3 > 0$

$(2x+1)(x-3) > 0$ M1

CVs $x = -\frac{1}{2}$ $x = 3$ A1



M1

$x < -\frac{1}{2}$ or $x > 3$ A1

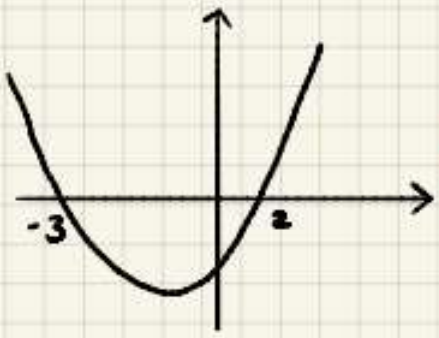
3. $x(2x+1) \leqslant x^2 + 6$

$2x^2 + x \leqslant x^2 + 6$ M1

$x^2 + x - 6 \leqslant 0$

$(x+3)(x-2) \leqslant 0$ M1

CVs $x = -3$ $x = 2$ A1



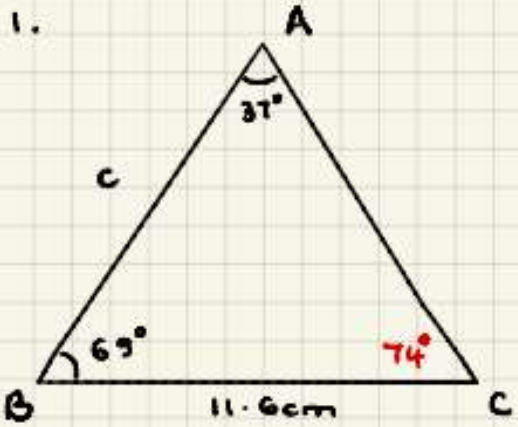
M1

$-3 \leqslant x \leqslant 2$ A1

III

E1 Triangle Geometry

1.

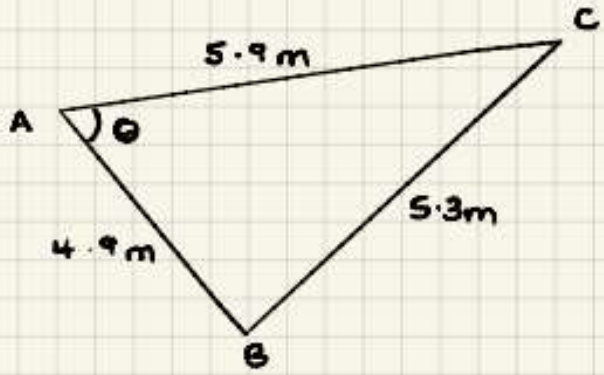


$$\frac{c}{\sin 74^\circ} = \frac{11.6}{\sin 37^\circ}$$

$$c = \frac{11.6 \sin 74^\circ}{\sin 37^\circ} \quad \text{M1}$$

$$c = 18.5\text{cm} \quad \text{A1}$$

2.



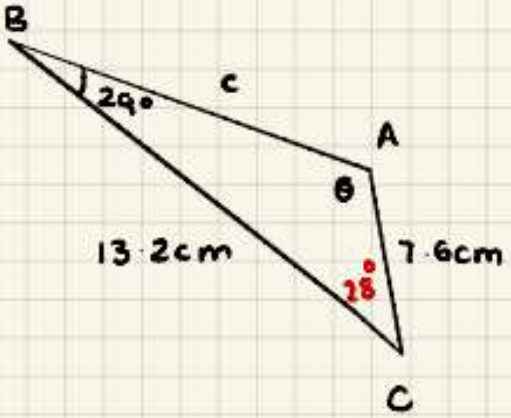
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{5.9^2 + 4.9^2 - 5.3^2}{2(5.9)(4.9)} \quad \text{M1}$$

$$\cos\theta = 0.53148$$

$$\theta = 57.9^\circ \quad \text{A1}$$

3.



θ

$$\frac{\sin\theta}{13.2} = \frac{\sin 29}{7.6}$$

$$\sin\theta = \frac{13.2 \sin 29^\circ}{7.6} \quad \text{M1}$$

$$\sin\theta = 0.8420$$

$$\theta = 57.4^\circ$$

obtuse $\Rightarrow \theta = 123^\circ$ A1

AB

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 13.2^2 + 7.6^2 - 2(13.2)(7.6)\cos 28 \quad \text{M1}$$

$$c^2 = 54.8 \Rightarrow c = 7.4\text{cm} \quad \text{A1}$$

Rearranging equations

1. Make x the subject of $x + a = \frac{x+b}{c}$ →

$$c(x+a) = x+b$$
$$cx + ca - x = b$$
$$cx - x = b - ca$$
$$x(c-1) = b - ca$$
$$x = \frac{b-ca}{c-1}$$

2. Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$ →

$$y(1-a) = x(1+a)$$
$$y - ay = x + xa$$
$$y - x = xa + ay$$
$$a(x+y) = y - x$$
$$a = \frac{y-x}{x+y}$$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \leq \theta \leq 360°$ giving your answers to 1 decimal place.

1.

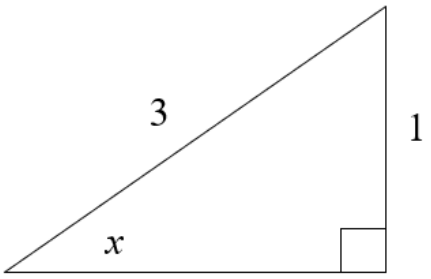
$$\theta = 66.4,\ 360 - 66.4$$
$$\theta = 66.4°,\ 293.6°$$

2.

$$2x = 30,\ 180 - 30,$$
$$360 + 30,\ 540 - 30$$
$$= 30,\ 150,\ 390,\ 510$$
$$x = 15,\ 75,\ 195,\ 255$$

E3 Exact trigonometric values

1.



Now Pythagoras's Theorem tells you that the third, adjacent, side is 3 $\sqrt{3^2 - 1^2} = \sqrt{8}$

Hence using SOH, CAH, TOA, $\cos x° = \frac{\sqrt{8}}{3}$ or $\frac{2\sqrt{2}}{3}$

$$\tan x° = \frac{\sqrt{2}}{4}$$