Summer Independent Learning:

A level Mathematics

Year 12 into Year 13

Part 1 – Compulsory

Complete the three practice papers over the summer. This will be more effective if you space them out over the break (e.g. one paper every 2-3 weeks).

For each paper:
- Attempt under timed conditions – **2 hours** per paper.
- Use the solutions on Sharepoint (Review DIL folder) to mark and correct your work in a different colour.
- Record your scores in the tables below.
- Identify **two** spec areas (e.g. E1, G4) that you need to work on.
- For these areas, watch the relevant Jack Brown videos (https://sites.google.com/view/tlmaths/home/a-level-maths/full-a-level) and complete practice questions from the Independent Practice sections of the class notes; or Dr Frost; or Madas Maths; or Sharepoint "Exam Questions by Topic".

Bring this completed record sheet together with all the work you have completed to your first lesson in September.

Paper 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Score	6	3	4	5	4	7	4	5	6	7	6	8	5	13	12	6	101

Spec area 1:	JB videos watched?	Practice completed?
Spec area 2:	JB videos watched?	Practice completed?

Paper 2

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Score	6	8	4	13	5	5	5	6	6	5	10	11	9	6	99

Spec area 1:	JB videos watched?	Practice completed?
Spec area 2:	JB videos watched?	Practice completed?

Paper 3

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Score	7	7	11	9	4	5	5	7	6	9	4	10	12	9	105

Spec area 1:	JB videos watched?	Practice completed?
Spec area 2:	JB videos watched?	Practice completed?

<u>Part 2 – Optional additional revision</u>

Complete the two extension papers and mark using the solutions on Sharepoint.

Record your scores and make a note of any questions you would like to ask.

Extension Paper 1

Qu	1	2	3	4	5	6	7	8	Total
Score	5	9	8	12	10	6	5	8	63
Questions / Learning points / comments:									

Extension Paper 2

Qu	1	2	3	4	5	6	7	Total
Score	3	9	8	9	9	6	14	58
Questions / Learning points / comments:								

Practice Paper 1 (101 marks)

1. *E5, E7*	Find, to 1 decimal place, the values of θ in the interval $0 \leqslant \theta \leqslant 180°$ for which $$4\sqrt{3}\sin(3\theta + 20°) = 4\cos(3\theta + 20°).$$ **(Total 6 marks)**
2. *J3*	Find in exact form the unit vector in the same direction as $\mathbf{a} = 4\mathbf{i} - 7\mathbf{j}$. **(Total 3 marks)**
3. *G1*	Prove, from first principles, that the derivative of $5x^3$ is $15x^2$. **(Total 4 marks)**
4. *G3*	$$f(x) = x^3 - 4x^2 - 35x + 20.$$ Find the set of values of x for which f(x) is increasing. **(Total 5 marks)**
5. *B7 AL*	Find the exact solutions of the equation $\lvert 6x - 1\rvert = \lvert x - 1\rvert$. [4]
6. *G4*	**(a)** Find the exact value of the x-coordinate of the stationary point of the curve $y = x\ln x$. [4] **(b)** The equation of a curve is $y = \dfrac{4x + c}{4x - c}$, where c is a non-zero constant. Show by differentiation that this curve has no stationary points. [3]
7. *H5*	Show that $\displaystyle\int_2^8 \frac{3}{x}\,dx = \ln 64$. [4]
8. *H5*	Find **(i)** $\int 8e^{-2x}\,dx$, **(ii)** $\int (4x+5)^6\,dx$. **[5]**
9. *G4*	Find $\dfrac{dy}{dx}$ in each of the following cases: **(i)** $y = x^3e^{2x}$, **[2]** **(ii)** $y = \ln(3 + 2x^2)$, **[2]** **(iii)** $y = \dfrac{x}{2x+1}$. **[2]**
10. *G4*	Find the equation of the normal to the curve $y = \dfrac{x^2 + 4}{x + 2}$ at the point $(1, \frac{5}{3})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

11. *E5 AL*

Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)

12. *H6*

(*a*) Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(*b*) Hence find the exact value of $\displaystyle\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\,dx$, giving your answer as a single logarithm.

(5)

13. *D1 AL*

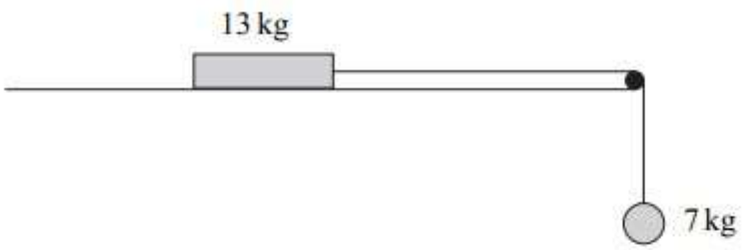
$$f(x) = (2-5x)^{-2}, \quad |x| < \tfrac{2}{5}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction.

(5)

14. *R4, R6*

The diagram shows a block, of mass 13 kg, on a rough horizontal surface. It is attached by a string that passes over a smooth peg to a sphere of mass 7 kg, as shown in the diagram.



The system is released from rest, and after 4 seconds the block and the sphere both have speed $6\,\text{m s}^{-1}$, and the block has **not** reached the peg.

(a) State **two** assumptions that you should make about the string in order to model the motion of the sphere and the block. *(2 marks)*

(b) Show that the acceleration of the sphere is $1.5\,\text{m s}^{-2}$. *(2 marks)*

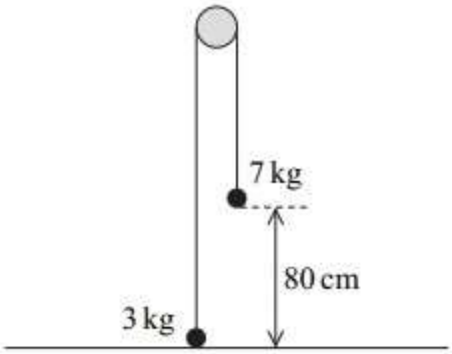
(c) Find the tension in the string. *(3 marks)*

(d) Find the coefficient of friction between the block and the surface. *(6 marks)*

15.

R4

Two particles, of masses 3 kg and 7 kg, are connected by a light inextensible string that passes over a smooth peg. The 3 kg particle is held at ground level with the string above it taut and vertical. The 7 kg particle is at a height of 80 cm above ground level, as shown in the diagram.



The 3 kg particle is then released from rest.

(a) By forming two equations of motion, show that the magnitude of the acceleration of the particles is $3.92\ \text{m s}^{-2}$.

[5 marks]

(b) Find the speed of the 7 kg particle just before it hits the ground.

[3 marks]

(c) When the 7 kg particle hits the ground, the string becomes slack and in the subsequent motion the 3 kg particle does not hit the peg.

Find the maximum height of the 3 kg particle above the ground.

[4 marks]

16.

R3

A crane is used to lift a crate, of mass 70 kg, vertically upwards. As the crate is lifted, it accelerates uniformly from rest, rising 8 metres in 5 seconds.

(a) Show that the acceleration of the crate is $0.64\ \text{m s}^{-2}$. *(2 marks)*

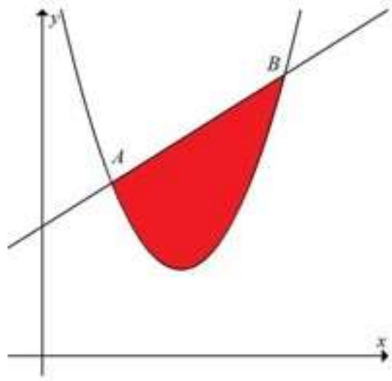
(b) The crate is attached to the crane by a single cable. Assume that there is no resistance to the motion of the crate.

Find the tension in the cable. *(3 marks)*

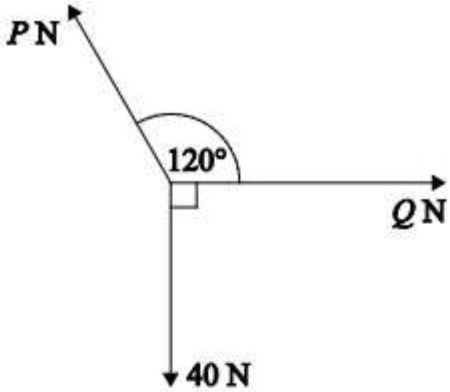
(c) Calculate the average speed of the crate during these 5 seconds. *(1 mark)*

Practice Paper 2 (99 marks)

1. *F4*	$\log_{11}(2x-1) = 1 - \log_{11}(x+4)$. Find the value of x showing detailed reasoning. **(Total 6 marks)**
2. *J2, R2*	A particle P of mass 6 kg moves under the action of two forces, F_1 and F_2, where $F_1 = (8\mathbf{i} - 10\mathbf{j})$ N and $F_2 = (p\mathbf{i} + q\mathbf{j})$ N, p and q are constants. The acceleration of P is $\mathbf{a} = (3\mathbf{i} - 2\mathbf{j})$ m s^{-2}. (*a*) Find, to 1 decimal place, the angle between the acceleration and **i**. **(2)** (*b*) Find the values of p and q. **(3)** (*c*) Find the magnitude of the resultant force R of the two forces F_1 and F_2. Simplify your answer fully. **(3)** **(Total 8 marks)**
3. *B9*	(*a*) Sketch the graph of $y = 8^x$ stating the coordinates of any points where the graph crosses the coordinate axes. **(2)** (*b*) (i) Describe fully the transformation which transforms the graph $y = 8^x$ to the graph $y = 8^{x-1}$. **(1)** (ii) Describe the transformation which transforms the graph $y = 8^{x-1}$ to the graph $y = 8^{x-1} + 5$. **(1)** **(Total 4 marks)**

4. H3	The diagram shows part of curve with equation $y = x^2 - 8x + 20$ and part of the line with equation $y = x + 6$.  (a) Using an appropriate algebraic method, find the coordinates of A and B. **(4)** The x-coordinates of A and B are denoted x_A and x_B respectively. (b) Find the exact value of the area of the finite region bounded by the x-axis, the lines $x = x_A$ and $x = x_B$ and the line AB. **(2)** (c) Use calculus to find the exact value of the area of the finite region bounded by the x-axis, the lines $x = x_A$ and $x = x_B$ and the curve $y = x^2 - 8x + 20$. **(5)** (d) Hence, find, to one decimal place, the area of the shaded region enclosed by the curve $y = x^2 - 8x + 20$ and the line AB. **(2)** **(Total 13 marks)**
5. G4	Find the equation of the tangent to the curve $y = \sqrt{4x+1}$ at the point (2, 3). [5]
6. B7 AL	Solve the inequality $\|2x - 3\| < \|x + 1\|$. [5]
7. H5	Given that $\int_0^a (6e^{2x} + x)\,dx = 42$, show that $a = \frac{1}{2}\ln\left(15 - \frac{1}{6}a^2\right)$. [5]
8. G4	Find, in the form $y = mx + c$, the equation of the tangent to the curve $$y = x^2 \ln x$$ at the point with x-coordinate e. [6]

9. *E5* *AL*	Solve, for $0 \leqslant \theta < 180°$, the equation $$2\cot^2\theta - 9\,\text{cosec}\,\theta = 3,$$ giving your answers to 1 decimal place. **(6)**
10. *D1* *AL*	$$f(x) = (3 + 2x)^{-3}, \quad \lvert x \rvert < \tfrac{3}{2}.$$ Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3. Give each coefficient as a simplified fraction. **(5)**
11. *H6*	$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$ (*a*) Find the values of the constants A, B and C. **(4)** (*b*) (i) Hence find $\int f(x)\ dx$. **(3)** (ii) Find $\int_0^2 f(x)\ dx$ in the form ln k, where k is a constant. **(3)**
12. *R6*	A block, of mass 5 kg, slides down a rough plane inclined at 40° to the horizontal. When modelling the motion of the block, assume that there is no air resistance acting on it. (a) Draw and label a diagram to show the forces acting on the block. *(1 mark)* (b) Show that the magnitude of the normal reaction force acting on the block is 37.5 N, correct to three significant figures. *(2 marks)* (c) Given that the acceleration of the block is $0.8\,\text{m s}^{-2}$, find the coefficient of friction between the block and the plane. *(6 marks)* (d) In reality, air resistance does act on the block. State how this would change your value for the coefficient of friction and explain why. *(2 marks)*

13. *R4*	A car of mass 1600 kg tows a trailer of mass 400 kg on a straight horizontal road. The car starts from rest and accelerates uniformly. The car travels 45 metres in 12 seconds. **(a)** Find the acceleration of the car. **[3 marks]** **(b)** A resistance force of magnitude 500 newtons acts on the car, and a resistance force of magnitude 80 newtons acts on the trailer. The trailer is connected to the car by a horizontal tow bar. A driving force of magnitude P newtons acts on the car. **(i)** Find the tension in the tow bar. **[3 marks]** **(ii)** Find P. **[3 marks]**
14. *R5*	Three forces, of magnitude 40 N, P N and Q N, all act in a horizontal plane. These forces are in equilibrium. The diagram shows the forces.  **(a)** Find P. **[3 marks]** **(b)** Find Q. **[3 marks]**

Practice Paper 3 (105 marks)

1. *B4*

The line with equation $mx - y - 2 = 0$ touches the circle with equation $x^2 + 6x + y^2 - 8y = 4$.

Find the two possible values of m, giving your answersin exact form.

(Total 7 marks)

2. *J4*

Given that point A has the position vector $4\mathbf{i} + 7\mathbf{j}$ and point B has the position vector $10\mathbf{i} + q\mathbf{j}$, where q is a constant, find

(*a*) the vector $\overrightarrow{AB}$ in terms of q.

(2)

(*b*) Given further that $|\overrightarrow{AB}| = 2\sqrt{13}$, find the two possible values of q showing detailed reasoning in your working.

(5)

(Total 7 marks)

3. *G3*

A fish tank in the shape of a cuboid is to be made from 1600 cm^2 of glass.
The fish tank will have a square base of side length x cm, and no lid. No glass is wasted.
The glass can be assumed to be very thin.

(*a*) Show that the volume, $V\,\text{cm}^3$, of the fish tank is given by $V = 400x - \frac{x^3}{4}$.

(5)

(*b*) Given that x can vary, use differentiation to find the maximum or minimum value of V.

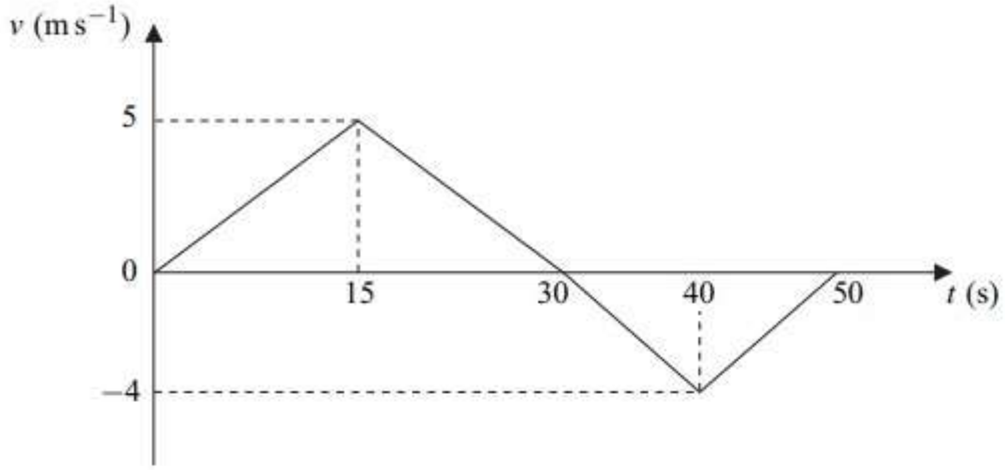
(4)

(*c*) Justify that the value of V you found in part **b** is a maximum.

(2)

(Total 11 marks)

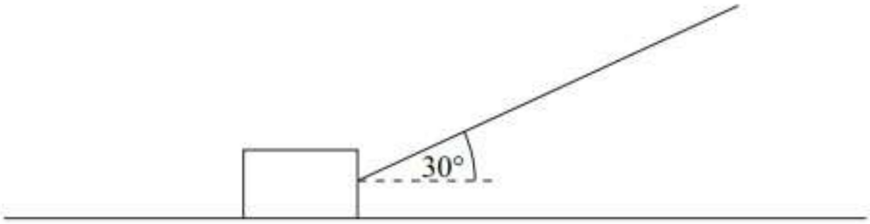
4. *C1*	The points A and B have coordinates $(3k - 4, -2)$ and $(1, k + 1)$ respectively, where k is a constant. Given that the gradient of AB is $-\frac{3}{2}$, (*a*) show that $k = 3$, **(2)** (*b*) find an equation of the line through A and B, **(3)** (*c*) find an equation of the perpendicular bisector of A and B. Leave your answer in the form $ax + by + c = 0$ where a, b and c are integers. **(4)** **(Total 9 marks)**
5. *H5*	Find the exact value of $\int_1^2 \frac{2}{(4x-1)^2}\,dx$. [4]
6. *G4*	Find the equation of the tangent to the curve $y = \frac{2x+1}{3x-1}$ at the point $(1, \frac{3}{2})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]
7. *B7* *AL*	Solve the inequality $\lvert 4x - 3 \rvert < \lvert 2x + 1 \rvert$. [5]
8. *G4*	For each of the following curves, find $\frac{dy}{dx}$ and determine the exact x-coordinate of the stationary point: **(i)** $y = (4x^2 + 1)^5$, **[3]** **(ii)** $y = \frac{x^2}{\ln x}$. **[4]**
9. *E5* *AL*	Solve, for $0 \leqslant \theta < 2\pi$, the equation $3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$ You must show all your working. Give your answers in terms of π. **(6)**

10. *D1* *AL*	(*a*) Expand $\dfrac{1}{\sqrt{(4-3x)}}$, where $\lvert x \rvert < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term. **(5)** (*b*) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x. **(4)**
11. *B10*	$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$ Find the values of the constants A, B and C. **(4)**
12. *Q2*	The graph shows how the velocity of a particle varies during a 50-second period as it moves forwards and then backwards on a straight line.  (a) State the times at which the velocity of the particle is zero. *(2 marks)* (b) Show that the particle travels a distance of 75 metres during the first 30 seconds of its motion. *(2 marks)* (c) Find the total distance travelled by the particle during the 50 seconds. *(4 marks)* (d) Find the distance of the particle from its initial position at the end of the 50-second period. *(2 marks)*

13.

R6

The diagram shows a block, of mass 20 kg, being pulled along a rough horizontal surface by a rope inclined at an angle of 30° to the horizontal.



The coefficient of friction between the block and the surface is μ. Model the block as a particle which slides on the surface.

(a) If the tension in the rope is 60 newtons, the block moves at a constant speed.

(i) Show that the magnitude of the normal reaction force acting on the block is 166 N. *(3 marks)*

(ii) Find μ. *(4 marks)*

(b) If the rope remains at the same angle and the block accelerates at 0.8 m s^{-2}, find the tension in the rope. *(5 marks)*

14.

R4, R6

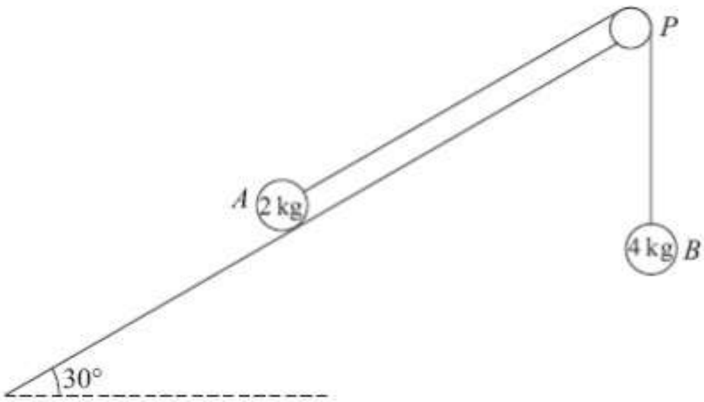


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

Extension Paper 1 (63 marks)

1. *G4*	$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$ (a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found. **(4)** (b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$ **(1)**
2. *D4*	An arithmetic sequence has first term a and common difference d. The sum of the first 36 terms of the sequence is equal to the square of the sum of the first 6 terms. **(a)** Show that $4a + 70d = 4a^2 + 20ad + 25d^2$ **[4 marks]** **(b)** Given that the sixth term of the sequence is 25, find the smallest possible value of a. **[5 marks]**
3. *C2*	Three points *A*, *B* and *C* have coordinates *A* (8, 17), *B* (15, 10) and *C* (−2, −7) **(a)** Show that angle *ABC* is a right angle. **[3 marks]** **(b)** *A*, *B* and *C* lie on a circle. **(b) (i)** Explain why *AC* is a diameter of the circle. **[1 mark]** **(b) (ii)** Determine whether the point *D* (−8, −2) lies inside the circle, on the circle or outside the circle. Fully justify your answer. **[4 marks]**

4.

D1 AL

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of $\dfrac{1}{\sqrt{4+x}}$

[3 marks]

(b) Hence, find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^3}}$

[2 marks]

(c) Using your answer to part **(b)**, find an approximation for $\int_0^1 \dfrac{1}{\sqrt{4-x^3}}\,dx$, giving your answer to seven decimal places.

[3 marks]

(d) (i) Edward, a student, decides to use this method to find a more accurate value for the integral by increasing the number of terms of the binomial expansion used.
Explain clearly whether Edward's approximation will be an overestimate, an underestimate, or if it is impossible to tell.

[2 marks]

(d) (ii) Edward goes on to use the expansion from part **(b)** to find an approximation

for $\int_{-2}^0 \dfrac{1}{\sqrt{4-x^3}}\,dx$

Explain why Edward's approximation is invalid.

[2 marks]

5.

B6

$p(x) = 30x^3 - 7x^2 - 7x + 2$

(a) Prove that $(2x+1)$ is a factor of $p(x)$

[2 marks]

(b) Factorise $p(x)$ completely.

[3 marks]

(c) Prove that there are no real solutions to the equation

$$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$$

[5 marks]

6.

G1

A curve has equation $y = x^3 - 48x$

The point *A* on the curve has x coordinate -4

The point *B* on the curve has x coordinate $-4 + h$

(a) Show that the gradient of the line *AB* is $h^2 - 12h$

[4 marks]

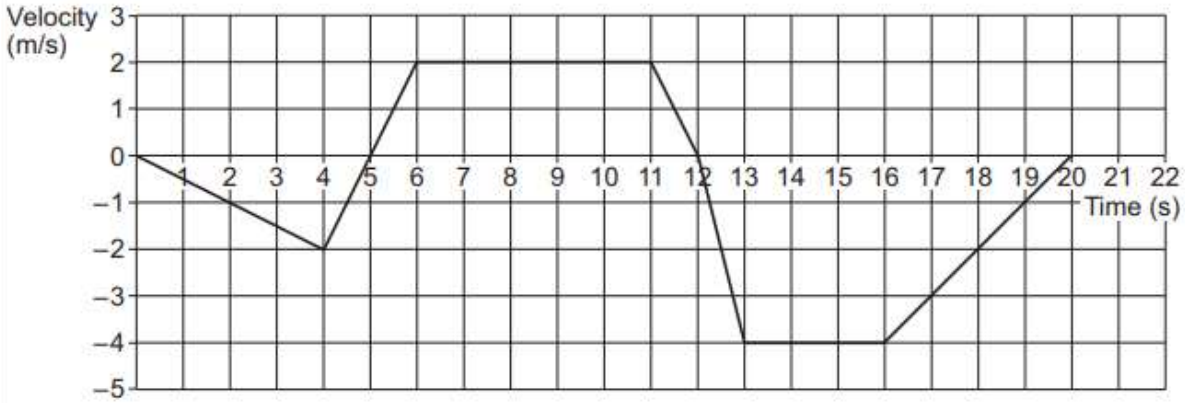
(b) Explain how the result of part **(a)** can be used to show that *A* is a stationary point on the curve.

[2 marks]

7.

Q2

The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



(a) Find the maximum magnitude of the acceleration of the object.

[1 mark]

(b) The object is at its starting position at times 0, t_1 and t_2 seconds.

Find t_1 and t_2

[4 marks]

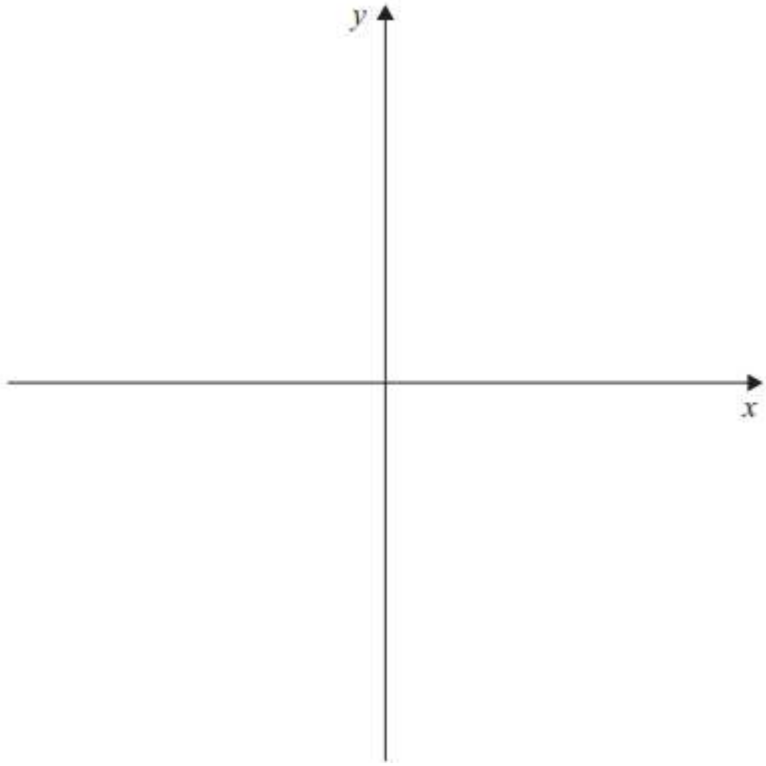
8.		**In this question use $g = 9.8\,\mathrm{m\,s^{-2}}$**
R6		A boy attempts to move a wooden crate of mass 20 kg along horizontal ground. The coefficient of friction between the crate and the ground is 0.85
	(a)	The boy applies a horizontal force of 150 N. Show that the crate remains stationary. **[3 marks]**
	(b)	Instead, the boy uses a handle to pull the crate forward. He exerts a force of 150 N, at an angle of 15° above the horizontal, as shown in the diagram.

Determine whether the crate remains stationary.

Fully justify your answer.

[5 marks]

Extension Paper 2 (58 marks)

1. *B7* *AL*		Sketch the graph of $y = \|2x + a\|$, where a is a positive constant. Show clearly where the graph intersects the axes. **[3 marks]** 
2. *C2*		Circle C_1 has equation $x^2 + y^2 - 8x - 14y = -40$ Circle C_2 has equation $(x - 16)^2 + (y - 12)^2 = 49$
	(a)	Determine whether C_1 and C_2 intersect. **[7 marks]**
	(b)	Find the maximum distance between a point on C_1 and a point on C_2. **[2 marks]**
3. *E5* *AL*	**(a)**	Prove the identity $\dfrac{\cos x}{\sec x + 1} + \dfrac{\cos x}{\sec x - 1} \equiv 2\cot^2 x$ **[3 marks]**

(b) Hence, solve the equation

$$\frac{\cos\left(2\theta+\frac{\pi}{3}\right)}{\sec\left(2\theta+\frac{\pi}{3}\right)+1}=\cot\left(2\theta+\frac{\pi}{3}\right)-\frac{\cos\left(2\theta+\frac{\pi}{3}\right)}{\sec\left(2\theta+\frac{\pi}{3}\right)-1}$$

in the interval $0 \le \theta \le 2\pi$, giving your values of θ to three significant figures where appropriate.

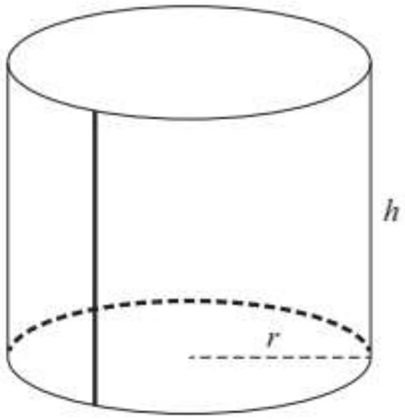
[5 marks]

4.

G3

Rakti makes open-topped cylindrical planters out of thin sheets of galvanised steel.

She bends a rectangle of steel to make an open cylinder and welds the joint. She then welds this cylinder to the circumference of a circular base.



The planter must have a capacity of 8000 cm^3

Welding is time consuming, so Rakti wants the total length of weld to be a minimum.

Calculate the radius, r, and height, h, of a planter which requires the minimum total length of weld.

Fully justify your answers, giving them to an appropriate degree of accuracy.

[9 marks]

5.

H6

(a) Express $\frac{5x^2-19x+50}{(1+3x)(5-x)^2}$ in the form $\frac{P}{1+3x}+\frac{Q}{5-x}+\frac{R}{(5-x)^2}$

where P, Q and R are constants.

[5 marks]

(b) Hence find $\int \frac{5x^2-19x+50}{(1+3x)(5-x)^2}\,dx$.

[4 marks]

6. J4	A quadrilateral has vertices A, B, C and D with position vectors given by $\overrightarrow{OA} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$, $\overrightarrow{OB} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}$, $\overrightarrow{OC} = \begin{bmatrix} 0 \\ 7 \\ 6 \end{bmatrix}$ and $\overrightarrow{OD} = \begin{bmatrix} 4 \\ 10 \\ 0 \end{bmatrix}$ **(a)** Write down the vector $\overrightarrow{AB}$ **[1 mark]** **(b)** Show that $ABCD$ is a parallelogram, but not a rhombus. **[5 marks]**
7. R4	A buggy is pulling a roller-skater, in a straight line along a horizontal road, by means of a connecting rope as shown in the diagram.  The combined mass of the buggy and driver is 410 kg A driving force of 300 N and a total resistance force of 140 N act on the buggy. The mass of the roller-skater is 72 kg A total resistance force of R newtons acts on the roller-skater. The buggy and the roller-skater have an acceleration of $0.2\,\text{m s}^{-2}$ **(a) (i)** Find R. **[3 marks]** **(a) (ii)** Find the tension in the rope. **[3 marks]** **(b)** State a necessary assumption that you have made. **[1 mark]** **(c)** The roller-skater releases the rope at a point A, when she reaches a speed of $6\,\text{m s}^{-1}$ She continues to move forward, experiencing the same resistance force. The driver notices a change in motion of the buggy, and brings it to rest at a distance of 20 m from A. **(c) (i)** Determine whether the roller-skater will stop before reaching the stationary buggy. Fully justify your answer. **[5 marks]** **(c) (ii)** Explain the change in motion that the driver noticed. **[2 marks]**